Plasma rotation in the MAST and JET tokamaks

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Abstract

Transport in tokamak fusion plasmas is predominantly driven by turbulence (micro-instabilities), while the presence of macro-scale MagnetoHydroDynamic instabilities (MHD) can cause confinement degradation or plasma disruption. Theory indicates that toroidal plasma rotation can suppress turbulence and stabilise MHD modes. Experimental studies were therefore carried out to characterise the actual rotation of tokamak plasmas and study its interaction with micro- and macro-instabilities. This thesis presents the database analysis of plasma rotation and confinement in JET and the investigation of the interaction between an ideal MHD mode and the plasma rotation in MAST.

The thermal Mach number in MHD-free JET plasmas is in the range $0.02 < M_{th} < 0.62$ and the Alfvén Mach number one order of magnitude lower. Observed trends and scaling laws reveal that Mach numbers decrease with increasing density and safety factor. Including the Alfvén Mach number in the dimensional scalings of momentum and energy confinement times significantly improves the scaling quality, highlighting the beneficial role of rotation in confinement. Dimensionless scalings of the confinement times are in agreement with results from dedicated experiments, while the positive scaling exponent of the Mach number is an indication of confinement enhancement with rotation.

Consistent theoretical, numerical and experimental analysis shows that the Long-Lived Mode (LLM) in MAST is an ideal saturated mode, unstable in plasmas with reversed or broad-low shear safety factor profiles. Whilst the mode is stabilised by toroidal flows, plasmas featuring the LLM exhibit strong core rotation damping and confinement degradation. The mode eigenstructure is calculated using the CASTOR code and its saturated amplitude is estimated from soft X-ray measurements. The LLM-induced braking according to Neoclassical Toroidal Viscosity (NTV) theory is calculated. The results, which cannot be explained by other damping mechanisms, are consistent with the observations, indicating that NTV theory provides a mechanism for the interaction between MHD and plasma rotation.
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Declaration

I declare that the work presented in this thesis, other than collaborative efforts stated in the acknowledgements, is my own.

Minh-Dúc HÚA, December 2009
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The work presented in this thesis is the result of vast collaborations. Chapter 4 presents the results of a database built under the direction of Dr Peter de Vries, with the common effort of Task Force T at JET. Dr Darren McDonald, Dr Carine Giroud and Dr Mike Johnson made particularly important contributions, while Thijs Versloot has now taken the responsibility to upgrade and maintain the database. Chapter 5 presents the analysis of MAST’s Long-Lived mode, which was lead by Dr Ian Chapman, with essential contributions of Dr Jim Hastie, Dr Simon Pinches and Dr Jon Graves. The study of the interaction of the mode with rotation is based on several diagnostic systems among which the Motional Stark Effect diagnostic (Dr Neil Conway, Dr Maarten de Bock, Dr Clive Michael), the Charge eXchange Recombination Spectroscopy system (Dr Neil Conway, Dr Marco Wisse), the Soft X-Ray fast cameras (Dr Luca Garzotti, Dr Alexey Zabolotskiy) and the Thomson Scattering system (Dr Mike Walsh, Dr Rory Scannel).
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Finally, I reserve my warmest thanks for my parents, who supported me in any circumstances and taught me how to lead anything I undertake with the best of my abilities and all available energy. These thanks are also for my fiancée, Túc, who always provided me with extra energy when I needed it, and made me convinced that the most beautiful things in life escape the rational laws of science.

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Original Publications

I. Lead-Author Journal Publications

M-D Hua, IT Chapman, SD Pinches, RJ Hastie, and the MAST team, submitted to Physical Review Letters, “Saturated internal instabilities in advanced tokamak plasmas”

M-D Hua, IT Chapman, AR Field, RJ Hastie, SD Pinches and the MAST Team, Plasma Physics and Controlled Fusion, 52, 035009 (2010), “Comparison of MHD-induced rotation damping with NTV predictions on MAST”

II. Contributing Author Journal Publications

IT Chapman, M-D Hua, SD Pinches, RJ Akers, AR Field, JP Graves, RJ Hastie, CA Michael and the MAST Team, accepted for publication in Nuclear Fusion, “Saturated ideal modes in advanced tokamak regimes in MAST”

PC de Vries, M-D Hua, DC McDonald, C Giroud, M Janvier, MF Johnson, T Tala, K-D Zastrow and JET EFDA Contributors, Nuclear Fusion, 48, 065006 (2008) “Scaling of rotation and momentum confinement in JET plasmas”


PC de Vries, TW Versloot, A Salmi, M-D Hua, DH Howell, C Giroud, V. Parail, G. Saibene, T Tala and JET EFDA Contributors, submitted to Plasma Physics and Controlled Fusion, “Momentum transport studies in JET H-mode discharges with an enhanced toroidal field ripple”
III. Lead Author Conference Contributed Presentation


Awarded the 2009 PPCF poster price

“On fait la science avec des faits comme une maison avec des pierres; mais une accumulation de faits n’est pas plus une science qu’un tas de pierres n’est une maison.”

“Science is built up with facts, as a house is with stones.; but a collection of facts is no more a science than a heap of stones is a house.”

Jules Henri Poincaré (1854-1912)
mathematician, physicist and philosopher
La Science et l’Hypothèse, 1902

May the research presented here meet my initial expectations: using theory to cement experimental observations, in an effort to build science with applications for mankind.
Chapter 1

Magnetic fusion

1.1 The energy issue

According to the International Energy Agency (IEA), the primary energy use of a British person in 2005 reached 125kWh or three quarter of an oil barrel (120L) per day [1,2]. This is the amount needed to boil 1400L of water, an equivalent of 6000 cups of tea. Societies, as they develop, have a tendency to become more and more reliant on energy, as reflected by the correlation between Human Development Index (HDI) and energy supply on figure 1.1-(a).

![Human Development Index vs Energy Supply](image1)

Figure 1.1: (a) The Human Development Index (HDI) as a function of daily primary energy supply per capita. The HDI is a measure of development including diverse factors such as education, health or wealth. Italy and Brazil are commonly considered as defining the upper and lower limits of the middle income countries. (source: UNDP). (b) The energy demand variation throughout the day for England and Wales in 2003-2004. The summer minimum demand is shown in dark blue, the typical summer one in light blue, the typical winter one in green, and the winter maximum one in red. (source: National Grid)
CHAPTER 1. MAGNETIC FUSION

<table>
<thead>
<tr>
<th></th>
<th>Proved reserves [units]/[10^9 bbl eq]</th>
<th>2005 production [units]/[10^9 bbl eq]</th>
</tr>
</thead>
<tbody>
<tr>
<td>oil [10^9 bbl]</td>
<td>1330/1330</td>
<td>30.8/30.8</td>
</tr>
<tr>
<td>coal [10^9 t]</td>
<td>844/4300</td>
<td>5.9/30.0</td>
</tr>
<tr>
<td>gas [10^{12} m^3]</td>
<td>175/1060</td>
<td>2.9/17.6</td>
</tr>
<tr>
<td>uranium [kt]</td>
<td>4700/446</td>
<td>68/6.5</td>
</tr>
<tr>
<td>total [10^9 bbl]</td>
<td>−/7136</td>
<td>−/84.9</td>
</tr>
</tbody>
</table>

Table 1.1: Proved reserves and production in 2005 by fuel source. The uranium oil equivalent is based on light water reactors, open cycle technology. The potential of renewable is very difficult to assess, and is therefore not shown here. (source: EIA)

Over the next decades, as more and more countries will achieve higher levels of development, the global energy demand is bound to increase drastically: scenarios from the IEA forecast a 50% increase in energy consumption by 2030, most of which will arise from non-OECD countries [1, 2].

The energy used nowadays is mainly generated by use of coal, gas, oil, nuclear, and various renewable production schemes, among which hydroelectricity, solar or wind sources. In contrast to the sharp growing demand, world resources are only available in finite quantities. Table 1.1 gives an account of the 2005 production and proved reserves for the main fuel types: oil, coal, gas and uranium. For these four fuels, the total reserves represent 85 times the amount of primary energy produced in 2005. This, however, does not mean that the world will run out of energy in about 80 years. Renewables, which potential is difficult to assess are not included here, nor is the influence of technological progress in fields such as fossil fuel extraction or fast neutron fission reactors. What is more, the energy demand remains a major unknown in this type of forecasts. This 85 years figure only illustrates that energy sources are rather scarce.

Scarcity is only one of many challenges posed by energy. In addition, energy resources are not evenly distributed among countries. This was shown by recent tensions around gas supply in central Europe, but the fact that 56% of the proved oil reserves are located in the Middle East, whereas a mere 1% is found on European ground, can serve as a more striking quantitative example [3]. As well as not equally abundant in space, energy is needed at precise times, and thus has to be produced, at precise times (figure 1.1-(b)), which is a major issue for intermittent energy production modes such as wind or solar. More generally, energy production faces many constraints fixed by the difficulty to store it, its transportation grid and the specific needs of its end-consumers. All of these factors require the energy generation infrastructure to be extremely flexible and reactive. An additional issue raised by energy production, and which has grown over the past decade, is environmental pollution, especially the one related to carbon dioxide emissions. Although not emitting any carbon dioxide, nuclear energy is confronted to problems such as long term radioactive waste, safety and proliferation risks.
Nuclear fusion could be part of the answer to this challenge. It consists in bringing two light nuclei together to form a heavier one, the process occurring with a release of energy. Fusion, as it is envisaged for energy production, relies on widespread isotopes of hydrogen: deuterium which can be extracted from seawater, and tritium, itself being obtained from lithium abundant in the Earth’s crust. Like conventional nuclear reactions used in energy production, it does not emit any carbon dioxide. But unlike them, it does not involve chain reactions, does not produce long term radioactive waste, nor has it potentially non-peaceful use.

Fuel scarcity, security of supply, limited storage possibilities, intermittency, environmental concern, safety and proliferation risks are as many complex characteristics of energy entailing that there will not be an ideal generation process able to support the entire production. Nevertheless, fusion appears to address most of these issues and can therefore be seen as a potential promising energy source to satisfy a major part of the demand.

1.2 Fusion

In the most accessible fusion reaction, a deuterium nucleus \((D)\) and a tritium one \((T)\) come together to form an alpha particle \(\left(4\text{He}\right)\) and a neutron \((n)\) according to the reaction:

\[
D + T \rightarrow 4\text{He} + n \tag{1.1}
\]

In order to fuse, the nuclei have to overcome the Coulomb potential barrier, and the reaction’s maximal cross section is reached for a temperature of about 100keV, a hundred times as hot as the sun’s core. Nevertheless, thanks to quantum mechanical tunneling and the high energy tail of the deuterium and tritium distributions, an optimal temperature for the reaction to happen is 20keV. The mass deficit in the products is 17.6MeV, 3.5MeV of which is carried by the alpha particle, and the remaining 14.1MeV by the neutron. Ideally, the alpha’s energy is used to sustain fusion reactions in the \(D-T\) mix, whereas the neutron is bound to escape the medium. This energy, however, is not lost as it can breed tritium from lithium \((Li)\):

\[
Li + n \rightarrow 4\text{He} + T \tag{1.2}
\]

At the temperature required for the reaction to take place, the fuel is an ionised gas called plasma. The collisions leading to the reaction can occur if a sufficient number of particles are confined for a sufficiently long time. A criterion was expressed by
CHAPTER 1. MAGNETIC FUSION

Figure 1.2: (a) Schematic of a tokamak and some geometrical quantities: the major and minor radii, the toroidal and poloidal angles, \( R_0, a, \phi \) and \( \theta \). (b) Different components creating the tokamak magnetic structure.

Lawson [4] in terms of:

\[
    n_e T_e \tau_E \geq 5 \times 10^{21} (\text{m}^{-3} \cdot \text{keV} \cdot \text{s})
\]  

(1.3)

Here, \( n_e \) is the electron number density, \( T_e \) the electron temperature and \( \tau_E \) the energy confinement time, the rate at which the plasma loses energy. Confinement can be achieved by having the charged particles of the plasma gyrate around the lines of magnetic fields: this is the aim of magnetic fusion.

1.3 Tokamaks

The most simple designs for confining magnetic devices are cylindrical, such as the Z- and \( \theta \)- pinches, where the fields are respectively azimuthal and axial. To reduce end losses these cylinders can be bent into a torus. The tokamak has such a shape, and is considered the most promising route towards achieving magnetic fusion. The overall shape of the tokamak can be described by the major radius, \( R_0 \), and minor radius, \( a \), (figure 1.2-(a)). The magnetic field wraps around the torus in a helical shape, and is therefore decomposed onto two directions, the toroidal and the poloidal directions, respectively described by the \( \phi \) and \( \theta \) angles (figure 1.2-(a)). The dominant part of the magnetic field is usually the toroidal part, generated through toroidal field coils (figure 1.2-(b)). The poloidal component is generated by a toroidal current, \( I_p \), driven through the plasma by means of a central solenoid. The surfaces tangent to the magnetic field, called flux surfaces, form a nested structure as shown on figure A.1 in appendix A.

The Mega Ampère Spherical Tokamak (MAST) and the Joint European Torus (JET) are two fusion experiments carried out at Culham, United Kingdom. As larger machines are expected to yield higher confinement, JET is currently the
largest existing tokamak, with $R_0 = 3\text{m}$ and a toroidal field $B_\phi = 4\text{T}$ (table 1.2), but also the closest in geometry to the next step machine, ITER.

In contrast to JET’s large size and tyre-like shape, MAST belongs to the rather more compact and cored-apple shaped Spherical Tokamak (ST) type (figure 1.3). STs are designed to be more cost effective, running at somewhat lower magnetic fields and not relying on super-conducting coils [5]. The fundamental shape difference with respect to a conventional tokamak can be described by the ratio of the minor and major radius, also called the inverse aspect ratio. It is used as a small parameter for asymptotic expansions in most of the theoretical work. Although valid for a conventional tokamak, this does not hold for an ST. A large inverse aspect ratio also means that the inboard/outboard asymmetry of the nested flux surface structure is considerably increased. Other differences include the facts that STs achieve higher ratios of volume-average kinetic to magnetic pressure, $\beta$, have a toroidal and poloidal field of same order of magnitude, and present a more elongated plasma cross-section (figure 1.3). Table 1.2 summarises the main characteristics of JET and MAST.

<table>
<thead>
<tr>
<th></th>
<th>JET</th>
<th>MAST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius ($R_0$[m])</td>
<td>3.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Minor radius ($a$[m])</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Inverse aspect ratio ($\epsilon$)</td>
<td>0.33</td>
<td>0.55</td>
</tr>
<tr>
<td>Central toroidal field ($B_{\phi,0}$[T])</td>
<td>4.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Plasma current ($I_p$[MA])</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Auxiliary heating power ($P_{\text{in}}$[MW])</td>
<td>37</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 1.2: Main characteristics of the JET and MAST tokamaks.
1.4 Motivation and outline

The aim of tokamak research is to develop the means of achieving thermonuclear fusion by confining the combustible plasma in a toroidal magnetic structure and heating it with different auxiliary systems. The plasma particles gyrate around the magnetic field lines, and move from one such line to another under the action of collisions and drift motions associated with the toroidicity of the device, leading to the radial transport of particles and energy. While this level of so-called “neoclassical” transport would possibly have been low enough to reach criteria like that described by Lawson (equation 1.3) in a machine such as JET, it is in practice not observed and several challenges have to be addressed in order to harness fusion as an energy source. From the confinement point of view, a major issue is the distortions of the magnetic structure, known as Magneto Hydro Dynamic (MHD) instabilities, leading to increased transport and potential disruption of the plasma. An additional source of concern is turbulence, which is the appearance of micro-scale fluctuations in the plasma, and is thought to be responsible for the observed transport at levels above the neoclassical predictions.

A tokamak plasma can rotate in the toroidal and poloidal directions. The poloidal flow is damped by dissipative mechanisms associated with the outboard-inboard asymmetry of the toroidal field. In contrast, the plasma can reach high toroidal velocity due to the strong torque resulting from the injection of high energy neutrals, with the aim of heating the plasma. This toroidal rotation is expected to enhance the plasma’s MHD stability and help suppress turbulence, meaning that rotation can play a major role in achieving fusion conditions in tokamaks. Although theories have studied the effects of toroidal flows rather extensively, much analysis remains to be done in order to experimentally characterise the actual rotation of tokamak plasmas and study its interaction with both micro- and macro-instabilities, while keeping a strong link between the observations and the previously accumulated theoretical insight. The work presented here is an attempt to contribute to meeting these objectives.

To do so, chapter 2 introduces the formalisms required to study a plasma. The emphasis is naturally laid on the theory of plasma rotation and its treatment as an angular momentum transport problem, with its torque source, transport and sinks. Some illustrations of the beneficial role of rotation in the suppression of MHD instabilities and turbulence are also provided. Chapter 3 details the measurement of plasma velocity and ion temperature by the charge exchange recombination spectroscopy diagnostic in MAST and JET. Broad trends of rotation in JET are presented in chapter 4, where the analysis of a large database is also used to identify the key plasma parameters influencing rotation. The confinement of energy and angular momentum is examined by means of scaling laws including rotation, in order to statistically assess the role of toroidal flows in confinement. The interpretation of these scalings should also provide indications about the relevant physics
models applicable to transport. Chapter 5 focuses on an MHD instability observed in MAST and thought to be relevant to a large range of tokamaks including the future machine ITER. The instability interacts with plasma flows, as it triggers a strong braking of the plasma and was shown to be potentially stabilised by high levels of toroidal rotation. The MHD instability itself is first investigated, which provides essential information to analyse the flow damping it produces. A theory suitable to explain such a damping, “Neoclassical Toroidal Viscosity” theory, is identified, its use is adapted to this specific case and the theoretical predictions are compared to the observed changes in plasma rotation. Lastly, chapter 6 draws general conclusions of the work presented.
Chapter 2

Plasma Physics and tokamak rotation

In order to investigate rotation in tokamak plasmas, it is necessary to describe this complex medium and identify the key physics involved. This chapter first details the essential characteristics of a high temperature fusion plasma and introduces formalisms used to represent its behaviour, spanning from the most detailed kinetic equations to the most simplified single fluid equations. The notions of magnetic flux and flux functions, central to the theory of magnetised plasmas, are presented together with the conditions of plasma equilibrium in a tokamak and of its stability. With these fundamentals explained, it is possible to tackle the governing equations of plasma rotation, from which it appears that a transport approach is best suited for this problem. This involves the angular momentum sources along with its transport and sinks, which can be accounted for by different mechanisms. Lastly, it is shown, with the help of simple physical pictures, how plasma rotation can benefit plasma confinement, hence justifying its in-depth study.

2.1 Basic plasma and tokamak physics

2.1.1 Introduction to plasmas

The behaviour of a plasma is dictated by its two fundamental characteristics: it is a gas and it is ionised. Because the particles of the plasma are ionised, they create an electromagnetic field as they move, which superimposes on the external field. Conversely, the motion of each particle is determined by the fields through the Lorentz force \( \vec{F} = m_\alpha (q_\alpha \vec{E} + \vec{v}_\alpha \times \vec{B}) \), where \( m \), \( q \) and \( \vec{v}_\alpha \) are the mass, charge and velocity of the particle, the \( \alpha \) subscript denoting its type. The total
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The electromagnetic field satisfies Maxwell’s equations:

\[ \vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \quad (\text{Gauss’ law}) \]  \hspace{1cm} (2.1)

\[ \vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (\text{Ampère’s law}) \]  \hspace{1cm} (2.2)

\[ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\text{Faraday’s law}) \]  \hspace{1cm} (2.3)

\[ \vec{\nabla} \cdot \vec{B} = 0 \]  \hspace{1cm} (2.4)

The particles do not only have to consistently obey these laws, they also interact with each other, which gives the plasma its fluid properties. In particular, the collisions occurring between the large numbers of electrons and ions allow the definition of corresponding temperatures and pressures as is possible in usual fluids (these quantities are defined in section 2.1.2). This thermal agitation plays a special role in a plasma: it screens the electrostatic interactions between particles beyond a certain length called the Debye length

\[ \lambda_D = \sqrt{\frac{\varepsilon_0 k T_e}{n_e e^2}} \]

The number of particles included in a sphere of radius \( \lambda_D \), \( N_D = \frac{4\pi}{3} \rho_{e} \lambda_D^3 \), is called the plasma parameter. To be considered a plasma, an ionised gas must feature this electrostatic shielding. This means that \( \lambda_D \) must be much shorter than the system’s characteristic length and that there should be enough particles in the Debye sphere for collective processes to happen there, \( N_D \gg 1 \). For a tokamak, \( T_e \sim 10\text{keV}, n_e \sim 10^{20}\text{m}^{-3} \) hence \( \lambda_D \sim 7\mu\text{m} \) and \( N_D \sim 2 \times 10^8 \).

2.1.2 Plasma description

In tokamak plasmas, the main species present are deuterium ions, simply referred to as ions, and electrons; impurities can be present in small amounts (of the order of the percent), the subscript \( \alpha \), denoting the particle species, is adapted accordingly in the equations. Underlying the plasma evolution is the Larmor gyration around the magnetic field lines, with a radius of \( \rho_{\text{L},\alpha} = \frac{m_{\alpha} v_{\perp}^2}{q_{\alpha} B} \) and a frequency \( \omega_{L,\alpha} = \frac{q_{\alpha} B}{m_{\alpha}} \). This motion due to electromagnetic forces is modified by collisions between particles in the plasma.

The most detailed plasma description is the kinetic one, which accounts for all types of particles in velocity space through the particles distribution function \( f_\alpha(\vec{r}, \vec{v}, t) \). Adopting this point of view, \( f_\alpha(\vec{r}, \vec{v}, t)d^3\vec{v}d^3\vec{r} \), denotes the number of particles with velocity between \( \vec{v} \) and \( \vec{v} + d\vec{v} \) located between \( \vec{r} \) and \( \vec{r} + d\vec{r} \) at the time \( t \). The plasma evolves according to the kinetic equation:

\[ \frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} f_\alpha = \frac{1}{m_\alpha} \left( q_\alpha \left( \vec{E} + \vec{v} \times \vec{B} \right) + \vec{F}_{\text{ext}} \right) \cdot \vec{\nabla} f_\alpha = \left( \frac{\partial f_\alpha}{\partial t} \right)_{\text{coll}} \]  \hspace{1cm} (2.5)
CHAPTER 2. PLASMA PHYSICS AND TOKAMAK ROTATION

There are as many kinetic equations as species in the plasma, each of them has an unknown \( f_\alpha \) of 7 independent variables \( (t, \vec{r} \text{ and } \vec{v}) \). Assuming a collision operator, \( \frac{\partial f_\alpha}{\partial t} \text{coll} \) enables to solve for the distribution function.

In the less detailed multi fluid description, the plasma is usually considered to be composed of the ion and electron fluids, but a fast ion fluid can sometimes be added. For each species, it is derived from the kinetic description by averaging the distribution function over the particle velocity, thus defining the fluid quantities:

\[
n_\alpha(\vec{r}) = \int f_\alpha(\vec{r}, \vec{v}) \, d^3\vec{v} \quad \text{(density)} \tag{2.6}
\]

\[
v_\alpha(\vec{r}) = \frac{1}{n_\alpha} \int f_\alpha(\vec{r}, \vec{v}) \vec{v} \, d^3\vec{v} \quad \text{(velocity)} \tag{2.7}
\]

\[
T_\alpha(\vec{r}) = \frac{m_\alpha}{n_\alpha} \int f_\alpha(\vec{r}, \vec{v})(\vec{v} - \vec{v}_\alpha)^2 \, d^3\vec{v} \quad \text{(temperature)} \tag{2.8}
\]

Then taking the corresponding moments of the kinetic equation gives the fluid equations governing the plasma fluids:

\[
\frac{\partial n_\alpha}{\partial t} + \vec{\nabla} \cdot \left( n_\alpha \vec{v}_\alpha \right) = 0 \tag{2.9}
\]

\[
m_\alpha n_\alpha \left( \frac{\partial \vec{v}_\alpha}{\partial t} + \left( \vec{v}_\alpha \cdot \vec{\nabla} \right) \vec{v}_\alpha \right) = -\vec{\nabla} p_\alpha - \vec{\nabla} \cdot \pi_\alpha + n_\alpha Z_\alpha e \left( \vec{E} + \vec{v}_\alpha \times \vec{B} \right) + \vec{R}_{\beta \neq \alpha} + \vec{f}_{\text{ext}} \tag{2.10}
\]

In equation 2.10, \( p_\alpha = n_\alpha T_\alpha \) is the scalar pressure and \( \pi_\alpha \) the viscous stress tensor. They arise from the departure of the velocity of individual particles from the collective velocity of the fluid, and their structure is discussed in more details in section 2.2.3. \( \vec{R}_{\beta \neq \alpha} \) and \( \vec{f}_{\text{ext}} \) respectively represent the interaction with other fluids in the plasma and the external volumic forces other than electromagnetic. Equations 2.9 and 2.10 are called the continuity and the force balance equations. There also exists an energy balance equation, not shown here, which is obtained by taking the second velocity moment of the kinetic equation.

Such a system describes one of the species in the plasma. For each of them, there are 4 independent variables but there is one unknown more than the number of equations. Solving the system thus requires an extra relation, which usually consists in assuming an equation of state for the corresponding fluid.

A further simplification can be made by removing the distinction between the ion and electron fluids to obtain a single fluid description. The single fluid quan-
tities are then defined as:
\begin{align}
n &= n_i = n_e \\
\rho &= (m_i + m_e) n \\
\bar{v} &= \frac{m_i \bar{v}_i + m_e \bar{v}_e}{m_i + m_e} \\
\bar{j} &= en (\bar{v}_i - \bar{v}_e)
\end{align}

Equation 2.11 simply comes from the plasma quasi-neutrality. Definition 2.13 states that the velocity of the single fluid is that of the centre of mass while the current density described by 2.14 is carried by the difference between the ion and electron velocities. Both species have a contribution on the collective behaviour of the single fluid, which is reflected by the definition of its temperature:
\begin{equation}
T = \frac{m_i}{n} \int f_i(\vec{r}, \vec{v})(\vec{v} - \bar{v})^2 d^3\vec{v} + \frac{m_e}{n} \int f_e(\vec{r}, \vec{v})(\vec{v} - \bar{v})^2 d^3\vec{v} \\
\simeq T_i + T_e
\end{equation}

In equation 2.15 the temperature is calculated using the single fluid velocity \( \bar{v} \) and not the ion and electron fluid velocity separately, \( \bar{v}_i \) and \( \bar{v}_e \), as in equation 2.8. The continuity balance equation for the single fluid is simply obtained by summing its ion and electron counterparts. Deriving the single fluid force balance equation requires relations similar to equation 2.10 for the ion and electron fluids, but where the pressure and stress are calculated using the single fluid velocity as a reference, like in equation 2.15. The electrostatic force vanishes and the interactions between fluids cancel, which yields the single fluid equations:
\begin{align}
\frac{\partial n}{\partial t} + \nabla \cdot (n \bar{v}) &= 0 \tag{2.16} \\
\rho \left( \frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla) \bar{v} \right) &= -\nabla p - \nabla \cdot \tau + \bar{j} \times \vec{B} + \vec{f}_{ext} \tag{2.17}
\end{align}

Here the pressure is defined as \( p = nT \). The single fluid description naturally has 4 independent variables and needs a closure relation, as did each of the ion and electron sets of equations in the two fluid description.

There are many different formalisms to treat the plasma behaviour, with various degrees of precision. The kinetic equations account for all particles of the plasma, while the information provided by the fluid description is comparatively reduced. This is done by defining an average velocity, the departure of the individual particles velocity from this average being described in terms of pressure and viscosity. The single fluid approach is usually sufficient for the investigation of the stability of the plasma. Rotation studies focus on the ion fluid, which velocity can be considered equal to that of the single fluid, because the electron mass is negligible compared to the ion mass.
2.1.3 Equilibrium

When in equilibrium, the magnetic configuration forms nested surfaces called flux surfaces, as mentioned in chapter 1. It is convenient to describe this equilibrium in a usual right-handed orthogonal cylindrical coordinate system with respect to the torus axis \((R, \phi, Z)\) using the local basis \((\vec{e}_R, \vec{e}_\phi, \vec{e}_Z)\) (figure A.1, appendix A). The innermost surface is a simple closed line called the magnetic axis. Due to the divergence free nature of the magnetic field, there exists a function \(\psi\) meeting the requirements:

\[
\vec{B} = \vec{B}_\phi + \frac{1}{R} \vec{\nabla} \psi \times \vec{e}_\phi
\]  

(2.18)

From this equation, keeping the axisymmetry of the field in mind, it can be seen that the magnetic field lies on toroidal surfaces of constant \(\psi\), which can therefore conveniently be used to label the nested flux surfaces. It can be proved that \(\psi\) is the flux of the poloidal field per unit toroidal angle through the horizontal surface containing the magnetic axis, leaning on the considered flux surface and oriented upwards. If the plasma current is oriented in the direction of increasing \(\phi\) then \(\psi\) is maximal on the magnetic axis and minimal at the plasma boundary. \(\psi\) can also be shown to be the toroidal component of the potential vector multiplied by the major radius, \(\psi = RA_\phi\).

The condition for equilibrium is given by the force balance equation 2.17. In a dissipationless, flowless and force free single fluid plasma, it can be written as:

\[
\vec{\nabla} p = \vec{j} \times \vec{B}
\]  

(2.19)

Such conditions are very rarely satisfied in plasmas. This situation can however be seen as equivalent to a steady state plasma with flows where viscosity is balanced by external torques (section 2.2). In cases where a flow is present, the centrifugal force acts on the plasma, but its effect on the equilibrium nested structure is shown to be negligible in plasmas with subsonic thermal Mach numbers of the order of \(10^{-1}\) as is the case in tokamaks [6]. More details about Mach numbers are given in section 4.3.2.

Complementary information on the current is given by Ampere’s law 2.2 assuming no displacement current. Substituting it into 2.19 and projecting onto the \(\vec{j}\) and \(\vec{\nabla} \psi\) directions respectively gives:

\[
RB_\phi = F(\psi)
\]  

(2.20)

\[
R \frac{\partial}{\partial R} \left( \frac{1}{R} \frac{\partial \psi}{\partial R} \right) + \frac{\partial^2 \psi}{\partial Z^2} = -\mu_0 R^2 \frac{\partial p}{\partial \psi} - \mu_0^2 F \frac{\partial F}{\partial \psi}
\]  

(2.21)

It can be proved that \(F(\psi) = I_{pol}/2\pi\), with \(I_{pol}\) the poloidal current passing through the surface \(Z = 0\) bounded by the flux surface. Equation 2.21 has \(\psi(R, Z)\)
CHAPTER 2. PLASMA PHYSICS AND TOKAMAK ROTATION

for unknown and is called the Grad-Shafranov equation. It is possible to solve it, hence deduce the shape of the flux surfaces, provided the pressure function \( p(\psi) \), current function \( F(\psi) \) and the boundary conditions on \( \psi \) and its first derivative, are known.

In MAST and JET, the EFIT [7] equilibrium code determines the most likely \( \psi(R,Z) \) function by means of fitting. The latter is based on information about the position of the plasma boundary, the vacuum magnetic field and data from various magnetic probes. A much more reliable reconstruction can be obtained by also including data from the Thomson Scattering (TS) and Motional Stark Effect (MSE) diagnostics to constrain the pressure and current functions.

The poloidal flux \( \psi \) is used in tokamak plasmas to label the flux surfaces to which the magnetic field is tangential. For convenience, the normalised poloidal flux is often used, it is defined as \( \psi_N = \frac{\psi - \psi_{axis}}{\psi_{boundary} - \psi_{axis}} \), spanning from 0 on the magnetic axis to 1 at the plasma boundary. \( \sqrt{\psi_N} \) is approximately proportional to the distance to the magnetic axis, and is therefore often used as well.

The basic equilibrium condition 2.19 guarantees that the pressure is constant on the flux surfaces, thus being called a flux function. The thermal transport along the flux surfaces is fast compared to that across them, hence the temperature is, to a good approximation, a flux function. In addition, due to the relation between the temperature and the pressure, \( p = nT \), the density can also be considered a flux function. Another flux function often used in tokamak physics is the safety factor \( q \) defined as the the flux surface average of the magnetic field’s inverse pitch angle. More concretely, \( q \) can be considered as the average number of toroidal turns a field line on this surface has to go through in order to complete a poloidal turn, thus representing the helicity of the field. It is worth mentioning that neither the electromagnetic field, nor the fluid velocity are flux functions.

In a rotating plasma however, the pressure is not a flux function anymore due to the centrifugal force. As mentioned earlier, this has little impact on the flux surfaces shape for low Alfvén Mach number. This effect on the pressure is purely linked to density: on a flux surface, the fluid is pushed towards the outboard side, while the rapid thermal transport still maintain a uniform temperature.

2.1.4 Stability

The equilibrium state where the \( \vec{j} \times \vec{B} \) force balances the pressure gradient can be perturbed. This equilibrium is unstable if a small departure from the initial state does not produce a restoring force that counteracts it. The question of equilibrium stability can be studied using ideal linear stability analysis. The equations
where equations 2.22, 2.23, 2.25, and 2.26 can respectively be recognised in equations 2.16, 2.17, 2.2, and 2.3. \( \gamma_c \) is the ratio of specific heats of the single fluid. Applying a small displacement \( \vec{\xi}, |\vec{\xi}|/a << 1 \), to the plasma results in a perturbation of the equilibrium quantities, \( X = X_0 + X_1 \) with \( X_1/X_0 << 1 \), and yields the linearised ideal MHD equations:

\[
\begin{align*}
\frac{\partial \rho_1}{\partial t} + \vec{v} \cdot (\rho_0 \vec{v}_1) &= 0 \quad (2.29) \\
\rho_0 \frac{\partial \vec{v}_1}{\partial t} + \vec{v} \cdot \vec{p}_1 - \vec{j}_1 \times \vec{B}_0 - \vec{j}_0 \times \vec{B}_1 &= 0 \quad (2.30) \\
\frac{\partial p_1}{\partial t} + \vec{v}_1 \cdot \vec{p}_0 + \gamma_c p_0 \vec{v}_1 \cdot \vec{v} &= 0 \quad (2.31) \\
\mu_0 \vec{j}_1 - \vec{v} \times \vec{B}_1 &= 0 \quad (2.32) \\
\frac{\partial \vec{B}_1}{\partial t} + \vec{v} \times \vec{E}_1 &= 0 \quad (2.33) \\
\vec{v} \cdot \vec{B}_1 &= 0 \quad (2.34) \\
\vec{E}_1 + \vec{v}_1 \times \vec{B}_0 &= 0 \quad (2.35)
\end{align*}
\]

In these equations, the equilibrium was taken to be flowless, as in section 2.1.3, resulting in \( \vec{v}_0 = \vec{0} \). The initial state is in equilibrium, such that \( \vec{\xi}(t=0) = \vec{0} \). It is of course possible to identify \( \vec{v}_1 \) with the time derivative of \( \vec{\xi} \). Eliminating \( \vec{j}_1 \) and \( \vec{E}_1 \), integrating equations 2.31 and 2.33, then substituting \( \vec{B}_1 \) and \( p_1 \) into equation 2.30 yields an initial value formulation of the ideal MHD linear stability problem:

\[
\rho_0 \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{F} \left( \vec{\xi} \right) \quad (2.36)
\]
where the force operator \( \vec{F} \) is defined by:

\[
\vec{F} = \frac{1}{\mu_0} \left[ \left( \vec{\nabla} \times \vec{\nabla} \times (\vec{\xi} \times \vec{B}_0) \right) \times \vec{B}_0 + \left( \vec{\nabla} \times \vec{B}_0 \right) \times \left( \vec{\nabla} \times (\vec{\xi} \times \vec{B}_0) \right) \right] + \vec{\nabla} \left( \gamma_c p_0 \vec{\nabla} \cdot \vec{\xi} + \vec{\xi} \cdot \vec{\nabla} p_0 \right)
\]  \hspace{1cm} (2.37)

It is only dependent on equilibrium quantities and linear in \( \vec{\xi} \). The second order differential equation 2.36 can be solved with initial conditions on \( \vec{\xi} \) and its first derivative, together with boundary conditions. It then provides the full evolution of the displacement, within the limits \( X_1/X_0 << 1 \) associated to the linearisation of the problem.

The initial value problem can be simplified by separating the time and space dependence of the displacement, noting that an \( e^{-i\Omega t} \) dependence is solution of equation 2.36, which then becomes:

\[-\rho_0 \Omega^2 \vec{\xi} = \vec{F} \left( \vec{\xi} \right) \]  \hspace{1cm} (2.38)

This is extremely convenient due to the linearity of \( \vec{F} \). Setting boundary conditions on \( \vec{\xi} \) and considering complex displacement fields defines a Hilbert space \( H \) of which \( \vec{F} \) is a linear operator \[8]. Solving equation 2.38 simply consists in diagonalising \( \vec{F} \) to find its eigenvectors (displacements multiplied by \( -\rho_0 \)) and eigenvalues (squared complex growth rates). The force operator was shown to be self-adjoint \( \left( \int \vec{\xi}_a^\ast \vec{F} \left( \vec{\xi}_b \right) d^3\vec{r} = \int \vec{\xi}_b^\ast \vec{F} \left( \vec{\xi}_a \right) d^3\vec{r} \right) \) for \( \vec{\xi}_a, \vec{\xi}_b \in H, \) \[9\]), meaning that its eigenvalues are real and that the plasma is unstable if one of them is negative. The instability occurring in the plasma is the eigenvector \( \vec{\xi} \) with the highest growth rate, corresponding to \( \min \Omega^2 \) for \( \Omega^2 \) eigenvalue of \( \vec{F} \). This explains why MHD perturbations are commonly referred to as MHD modes.

The self-adjointness of \( \vec{F} \) also allows the construction of a weak form of the eigenvalue formulation 2.38. This is obtained by multiplying equation 2.38 by \( \vec{\xi}^\ast \) (the complex conjugate of \( \vec{\xi} \)), dividing by 2 and integrating over the plasma volume:

\[-\frac{\Omega^2}{2} \underbrace{\int \rho_0 \left| \vec{\xi} \right|^2 d^3\vec{r}}_{K(\vec{\xi},\vec{\xi}^\ast)} - \frac{1}{2} \underbrace{\int \vec{\xi}^\ast \vec{F} \left( \vec{\xi} \right) d^3\vec{r}}_{\delta W(\vec{\xi},\vec{\xi}^\ast)} = 0 \]  \hspace{1cm} (2.39)

Here, \( K(\vec{\xi},\vec{\xi}^\ast) \) and \( \delta W(\vec{\xi},\vec{\xi}^\ast) \) can be identified as the kinetic and potential
energy of the displacement $\xi$. The following functional can then be defined:

$$\Omega^2(\xi, \xi^*) = \frac{\delta W(\xi, \xi^*)}{\frac{1}{2} \int \rho_0 \|\xi\|^2 d^3r}$$

Differentiating $\Omega^2(\xi, \xi^*)$ shows that the solutions of the eigenvalue formulation 2.38 are the stationary points of the functional. As a result, the eigenvector with the highest growth rate, that is the instability to occur in the plasma, is the displacement at which $\min \Omega^2$ is reached. Its growth rate is naturally given by $|\Omega|$.

This stability analysis is further simplified by the so-called energy principle [9]. Ideal MHD being dissipationless, considerations on the conservation of energy show that minimising $\delta W(\xi, \xi^*)$ rather than $\Omega^2(\xi, \xi^*)$ provides an estimate of the most unstable eigenvector and its growth rate. In any case, $K(\xi, \xi^*) > 0$ guarantees that minimising $\delta W(\xi, \xi^*)$ enables the prediction of the stability of the mode. The Energy Principle is thus the following: the equilibrium is unstable if and only if there exists a displacement with negative potential energy, in accordance with basic physical intuition.

$\delta W(\xi, \xi^*)$ can be modified into an expression easier to interpret by introducing $\vec{B}_1$ and $\vec{\kappa} = (\vec{B} \cdot \nabla) \vec{B}/B^2$ into it:

$$\delta W(\xi, \xi^*) = \frac{1}{2} \int \left( \frac{1}{\mu_0} \|\vec{B}_{1\perp}\|^2 + \frac{B_0^2}{\mu_0} \|\nabla \cdot \vec{\kappa} + 2\vec{\kappa} \cdot \vec{\xi} + 2\vec{\xi} \cdot \vec{\kappa} \|^2 \right) + \int \frac{\gamma_c p_0}{\mu_0} \|\nabla \cdot \vec{\xi}\|^2 \right) d^3r$$

$$+ \delta W_V(\xi, \xi^*)$$

Here, the $\perp$ subscript denotes the part perpendicular to the equilibrium magnetic field, and $\vec{\kappa}$ represents its curvature. $\delta W_V(\xi, \xi^*)$ is a term accounting for the interaction of the perturbation with the plasma-surrounding vacuum and is always stabilising, meaning positive. The first three terms can be interpreted respectively as the energies necessary to bend the field lines, compress them and compress the plasma, they are always stabilising. The fourth term can be destabilising, it is driven by the current, $\vec{j}_0$, and is often referred to as “kink term”. The fifth term is destabilising in regions where the pressure gradient is in the same direction as the curvature. It is called the “interchange term”, and its drive is the pressure gradient.
The variational normal mode approach and the energy principle are treatments of the linear MHD stability analysis which can be used analytically and numerically. Some MHD codes investigate stability in a slightly different manner, by directly solving the MHD equations 2.29-2.32 without the use of the force operator defined by equation 2.37. The CASTOR code [10] is a toroidal code solving equations 2.29-2.32. Taking into account equations 2.33, 2.34 and a resistive version of equation 2.35, the unknowns are the 8 scalar space functions linked to the perturbed mass density, velocity vector, vector potential and temperature. As such, the system to solve is a matrix equation of dimension 8, but where each matrix and vector coefficient is a scalar function of space, a differential operator or a combination of both. Such a formulation is simplified by decomposing the unknowns in Fourier space for the toroidal and poloidal direction, and Hermite elements $h_j(\sqrt{\psi N})$ for the radial direction. Their time dependence is assumed exponential, thus giving:

$$X_i(\sqrt{\psi N}, \phi, \theta_s) = e^{\gamma t} e^{in\phi} \sum_{j,m} c_{i,j,m} h_j(\sqrt{\psi N}) e^{im\theta_s}$$

(2.42)

Here, space is discretised in $N_s$ radial intervals and only the perturbation of toroidal periodicity $n$ is considered. The coordinates used are the straight field line coordinates, with the toroidal geometrical angle to locate a point toroidally, $(\sqrt{\psi N}, \phi, \theta_s)$ (appendix A, section A.1). The unknown vector is now $w$, a vector of dimension $16N_sM$, but each of its coefficient is simply a scalar, as opposed to a scalar function of space previously. There are however only 8 equations, meaning that the system is not solvable in this form. This is addressed by taking a weak form of equations 2.29-2.32, following the Galerkin method [10]. The latter consists in multiplying each equation by each of the $2N_sM$ base functions and integrating the result over space, effectively yielding $16N_sM$ equations from which differential operators are absent. The system can now be written in the form:

$$M_1 w = \gamma M_2 w$$

(2.43)

where $M_1$ and $M_2$ are square matrices of dimension $16N_sM$, and $M_2$ is hermitian definite positive. This is an eigenvalue problem, where the eigenvector with the eigenvalue of largest real part represents the instability occurring in the plasma. The special properties of $M_2$ enable this problem to be dealt with iteratively, based on an adaptation of the inverse iteration scheme for eigenvalue problems [11]. Denoting the Hermitian conjugate with the superscript $H$ and starting from an initial guess $(\gamma_0, w_0)$ for the eigenvalue and the eigenvector, the iteration defined
by:

\[
\begin{align*}
    w_{i+1} &= (\gamma_i - \gamma_0) (M_1 - \gamma_0 M_2)^{-1} M_2 w_i \\
    \gamma_{i+1} &= \frac{w_{i+1}^2 (M_1 - \gamma_0 M_2) w_i}{w_{i+1}^2 M_2 w_i + \gamma_0}
\end{align*}
\] (2.44)

converges to the eigenvector with eigenvalue closest to \(\gamma_0\). An appropriate choice of initial conditions therefore successfully analyses the linear MHD stability of the plasma.

## 2.2 Plasma rotation

In the previous section, various descriptions of a plasma were presented, as well as the structure of the magnetic field and physical quantities in this medium. This section deals with plasma rotation, the motion of the single fluid along the flux surfaces. It focuses on the toroidal component of this motion and aims at explaining the basics of rotation and the formalisms used to study it.

### 2.2.1 Governing equations

In a steady state plasma with no viscosity, inertia, nor external forces, the ion fluid force balance 2.10 can be expressed as:

\[
\nabla p_i = n_i Z_i e \left( \vec{E} + \vec{v}_i \times \vec{B} \right)
\] (2.45)

This relation simplistically assumes no interaction of the ion fluid with the electron one. Using the toroidal coordinates \((r, \phi, \theta)\) with the local orthonormal basis \((\vec{e}_r, \vec{e}_\phi, \vec{e}_\theta)\) shown on figure A.1, appendix A, it provides an estimate of the toroidal rotation:

\[
v_{i\phi} = \frac{\nabla p_i, \vec{e}_r}{n_i Z_i e B_\theta} - \frac{E_r}{B_\theta} + \frac{v_\theta}{B_\theta} B_\phi
\] (2.46)

It unveils the close interaction between the rotation, pressure gradient and electromagnetic field. Changes in any of these will therefore have an influence on plasma rotation.

Although this simple picture helps understand the link between rotation, electromagnetic and kinetic parameters of the plasma, the assumptions made are not verified in plasmas. First of all, external forces as well as viscosity must be taken into account. It is then best to consider the rotation of the single fluid in order to avoid tackling the intricate issue of the momentum transfer between the ion and electron fluid. Because of the predominance of the ion mass over that of the electron, the velocity and momentum of the single fluid can be considered equal.
to the velocity and momentum of the ion fluid. These quasi-equalities are used throughout this thesis.

The toroidal symmetry of the tokamak magnetic field provides a conserved quantity better suited to rotation studies than the linear momentum: the angular momentum. The angular momentum density is defined as:

\[ l_\phi (\psi) = m_i n (\psi) \langle R^2 \rangle (\psi) \omega (\psi) \]  

(2.47)

where \( \langle R^2 \rangle \) is the flux surface average squared major radius and \( \omega (\psi) = v_\phi / R \) the toroidal angular frequency. \( \omega \) is usually assumed to be a flux function, and as a result, \( l_\phi \) is flux function as well. This makes the angular momentum transport a one dimensional problem. The assumption that \( \omega \) is a flux function arises from the fact that to first order, in the absence of poloidal flow and in the conditions of equation 2.45, the angular frequency is given by the toroidal diamagnetic flow and the \( \vec{E} \times \vec{B} \) drift (the first two terms of equation 2.45):

\[ \omega (\psi) = -\frac{d\Phi}{d\psi} - \frac{1}{n_i Z_i e} \frac{d}{d\psi} \]  

(2.48)

Ohm’s law in an ideal plasma, \( \vec{E} + \vec{v} \times \vec{B} = -\vec{\nabla} \Phi + \vec{v} \times \vec{B} = 0 \), ensures that the electrostatic potential is a flux function, such that the angular frequency in equation 2.48 is itself a flux function.

An equation describing the angular momentum evolution could be derived by multiplying the force balance equation 2.10 for the ion fluid by the major radius \( R \) and averaging it over the flux surface. Such an equation would reveal that angular momentum is deposited in the plasma according to the torque density profile, transported under the action of inertia and viscosity and eventually lost at the plasma edge, thus justifying the adoption of a transport approach rather than a force balance approach. The radial angular momentum flux arising from transport and edge losses may be most simply described as an effective diffusion term, \( \vec{\Gamma}_{l_\phi} (\psi) = -\chi_\phi (\psi) \vec{\nabla} l_\phi (\psi) \), \( \chi_\phi \) being the angular momentum diffusivity. The use of this diffusive mechanism is influenced by the heuristic representation of transport in tokamak as a random walk process (section 2.2.3). Using this formalism, the local balance of momentum sources and sinks then takes the form:

\[ \frac{\partial l_\phi}{\partial t} = t_\phi - \vec{\nabla} \cdot \vec{\Gamma}_{l_\phi} \]  

(2.49)

where \( t_\phi \) is the torque density. Emphasis can be laid on the momentum contained within the volume enclosed by a flux surface, \( \int_{\psi_{axis}}^{\psi} l_\phi dV \). Integrating equation 2.49 from the magnetic axis to the corresponding flux surface, and using Stokes’
THEOREM, GIVES:

\[ \frac{\partial}{\partial t} \left( \int_{\psi_{\text{axis}}} \psi \, t_\phi dV \right) = \int_{\psi_{\text{axis}}} \psi \, t_\phi dV - \oint_{\psi' = \psi} \vec{\Gamma}_{l_\phi} (\psi').d\vec{S} \]  

(2.50)

\[ = \int_{\psi_{\text{axis}}} \psi \, t_\phi dV - \chi_\phi (\psi) \left\| \nabla l_\phi (\psi) \right\| \oint_{\psi' = \psi} e_r.d\vec{S} \]  

(2.51)

\[ = \int_{\psi_{\text{axis}}} \psi \, t_\phi dV - A (\psi) \left( \chi_\phi (\psi) \left\| \nabla l_\phi (\psi) \right\| \right) \]  

(2.52)

where \( A (\psi) \) is the area of the flux tube and \( \int_{\psi_{\text{axis}}} \psi \, t_\phi dV \) the torque applied to it. \( e_r \) is the radial unit vector of the toroidal coordinates described in section A.1. In steady-state, this yields:

\[ \chi_\phi (\psi) = \frac{\int_{\psi_{\text{axis}}} \psi \, t_\phi dV}{A (\psi) \left\| \nabla l_\phi (\psi) \right\|} \]  

(2.53)

This diffusive representation of momentum transport is most convenient in order to interpret tokamak experiments. The torque density \( t_\phi \) is usually calculated numerically using a Monte-Carlo simulation code such as TRANSPI [12], relying on measurements of plasma temperature and density. The gradient of angular momentum density \( \nabla l_\phi \) is calculated from measurements and the flux tube’s area \( A \) is given by an equilibrium reconstruction. This fully determines momentum diffusivity, which is the only unknown of equation 2.53.

The angular momentum flux can also be described as both a diffusive and a convective process, by means of a momentum diffusion coefficient and a momentum pinch velocity \( \vec{v}_p \):

\[ \vec{\Gamma}_{l_\phi} = -\chi_\phi \vec{\nabla} l_\phi + n\vec{v}_p l_\phi \]  

(2.54)

This representation of momentum transport arises from recent theories of turbulence [13]. Equation 2.54 has two unknowns, \( \chi_\phi \) and \( \vec{v}_p \) (the pinch velocity is in the radial direction), meaning that steady-state analysis of plasma experiments is not sufficient to determine the momentum diffusion and the momentum pinch velocity. The momentum pinch is therefore usually investigated using modulation experiments, whereby a varying torque is applied to the plasma, and both the amplitude of the plasma response and its phase relative to the torque modulation are analysed. Recent modulation experiments indicated that a radial angular momentum flux including not only a diffusive part but also a momentum pinch is best suited to accurately account for momentum transport [14, 15].

It should be noted that here, the emphasis is laid on the angular momentum flux. These conventions are chosen in order to deal with (approximate) flux functions. Other conventions can be used, in particular investigating the linear
momentum flux. This flux involves the plasma density and toroidal velocity, as well as their gradients. Nevertheless, the toroidal velocity is not a flux function, as mentioned earlier.

A scalar measure of momentum transport called momentum confinement time, $\tau_\phi$, can also be defined. Considering the total angular momentum of the plasma ($L_\phi$) and assuming the momentum losses are proportional to $L_\phi$ itself:

$$\left( \frac{\partial L_\phi}{\partial t} \right)_{\text{loss}} = -\frac{L_\phi}{\tau_\phi} \quad (2.55)$$

This simply implies that the momentum of the plasma is lost at a rate $\tau_\phi^{-1}$. With $T_\phi$ denoting the total torque, the conservation of momentum yields:

$$\tau_\phi = \frac{L_\phi}{T_\phi - \frac{\partial L_\phi}{\partial t}} \quad (2.56)$$

and in steady state, the momentum confinement time reduces to the simple relation:

$$\tau_\phi = \frac{L_\phi}{T_\phi} \quad (2.57)$$

The total angular momentum and momentum confinement time are of order $2 \times 10^{-2}$ N.m.s and 50ms in MAST, 5N.m.s and 200ms in JET.

### 2.2.2 Rotation sources

The first phenomenon involved in momentum transport is the deposition of angular momentum in the plasma according to the torque density profile. In a tokamak, torque can have several origins, among which are heating systems such as Neutral Beam Injection (NBI) and radio-frequency heating. In this section, only NBI torque will be discussed.

NBI heating consists in injecting high energy neutrals into the plasma. These fast particles then ionise and impart their energy to the plasma, resulting in a temperature increase. As they transfer their energy to the bulk plasma, fast particles also transfer their angular momentum, thus acting as a torque source. Figure 2.1 show the NBI system configurations for MAST and JET. NBI can be applied somewhat parallel to the plasma current, the so called co-injection, or in the opposite direction, known as counter-injection.

From angular momentum conservation considerations, it is possible to estimate the total NBI torque applied to the plasma. The linear momentum carried by a NBI fast particle is:

$$p_{\text{NBI}} = m_{\text{NBI}} v_{\text{NBI}} = \sqrt{2m_{\text{NBI}}E_{\text{NBI}}} \quad (2.58)$$
with $m_{\text{NBI}}$ the mass of the injected particles and $E_{\text{NBI}}$ their acceleration energy. The particles are injected at a rate equal to

$$\dot{N} = \frac{P_{\text{NBI}}}{E_{\text{NBI}}}$$

(2.59)

where $P_{\text{NBI}}$ is the heating power. The rate of angular momentum injection, meaning torque, is therefore given by:

$$T_\phi = \dot{N} R_{\text{NBI}} P_{\text{NBI}} = P_{\text{NBI}} R_{\text{NBI}} \sqrt{\frac{2m_{\text{NBI}}}{E_{\text{NBI}}}}$$

(2.60)

where $R_{\text{NBI}}$ is the injection tangency radius of the beam. $E_{\text{NBI}}$ and $R_{\text{NBI}}$ are of the order of 50keV and 1m for MAST and 100keV and 2m for JET giving a torque of order 1N.m per MW of power in both machines.

However, the torque deposition profile rather than simply the total torque is needed to study rotation. After entering the plasma, the beam neutrals do not interact much with the plasma before they are ionised. When this happens, they travel on the flux surface on which they were born, depositing their momentum (and energy) at the vicinity of this flux surface. As seen later in this section, the deposition of momentum by electromagnetic processes also modifies this profile. For beam energies used in tokamaks, the dominant process for beam particle ionisation is charge exchange reactions with plasma thermal ions, the mean free path of the neutral is thus $(n\sigma_{\text{CX, plasma}} - \text{beam})^{-1}$ giving a exponentially decreasing density of the beam neutrals:

$$n_{\text{NBI}} (l) = n_{\text{NBI}} (0) e^{\left(-\int_0^l n(\psi)\sigma_{\text{CX, plasma}} - \text{beam}(\psi)d\psi\right)}$$

(2.61)
In tokamaks, the mean free path is such that momentum and heat are deposited on axis. $\sigma_{CX,\text{plasma–beam}}$ increases with the background plasma density such that the torque deposition is broader for high density plasmas, whereas the opposite is true for low density plasmas.

The transfer of angular momentum to the plasma occurs on three different timescales, corresponding to three distinct mechanisms which consecutively strip the injected fast particles of their momentum to redistribute it to the plasma [16,17]. The first mechanism is electromagnetic and is due to the a rapid change in radius for the newly ionised particle.

In co-injection, a major part of particles ionised on the outboard side of the tokamak follow trapped drift orbit around a radius smaller than the ionisation radius. As a result, an apparent inward current sustained by the beam fast ions arises and has to be balanced by an outward displacement current in the plasma.
to preserve the plasma’s quasi-neutrality. A $\vec{j} \times \vec{B}$ force in the co-current direction is thus exerted on the plasma (figure 2.2-(a)), which deposits momentum on the fast timescale of the orbit bounce period ($\sim 10^{-5}$s).

In counter direction, the trapped orbit leads the ionised particles outwards and eventually lets it escape the plasma. In this case, the apparent current points outwards and the balancing displacement current inwards, in contrast to the co-current injection. Nonetheless, the plasma current, hence the poloidal field, also reverses direction as compared to the co-NBI case: the $\vec{j} \times \vec{B}$ remains in the co-NBI direction (2.2-(b)). The plasma therefore gains momentum, again on an orbit bounce period timescale.

It is also possible to describe this mechanism using the conservation of canonical angular momentum, which includes the momentum of the electromagnetic field by multiplying the usual linear canonical momentum by the major radius:

$$p_\phi = R (m_\alpha v_{\phi,\alpha} + q_\alpha A_\phi) = m_\alpha R v_{\phi,\alpha} + q_\alpha \psi$$

with the conventions of section 2.1.3. These mean that the plasma current is in the direction of increasing $\phi$, the NBI in that same direction for co-injection, $\psi$ maximal on the magnetic axis and minimal at the plasma edge. This approach has the advantage of being quantitative, as the variation in the fast particle momentum is given by:

$$\Delta L_{NBI} = -e \Delta \psi$$

hence:

$$\Delta L_\phi = -\Delta L_{NBI} = e \Delta \psi \begin{cases} \text{co-injection: } > 0 \text{ ie in beam direction} \\ \text{counter-injection: } < 0 \text{ ie in beam direction} \end{cases}$$

which describe the effective transfer of momentum from the NBI particles to the plasma through electromagnetic processes. This first deposition is called prompt torque.

This electromagnetic transfer completed, the fast particles carry on travelling around the torus, they collide on bulk ions and electrons, imparting their momentum to the plasma. This process is called collisional torque and occurs on a slowing-down timescale, of the order of tens of milliseconds in tokamaks. Once the energetic ions have slowed down to a velocity comparable to that of the thermal plasma, they are not considered as fast anymore and become part of the plasma, adding their momentum to the latter. This addition of momentum can be accounted for by a torque, which is called the thermalisation torque. Figure 2.3 gives a summary of the angular momentum transfers from the fast ions to the plasma.
NBI is not the only source of torque in a tokamak plasma. While presenting the mechanisms of NBI torque, the role of radial currents was emphasised through the prompt torque mechanism. Naturally, any radial current not originating from NBI would be a local source of torque, the direction of the drive depending on the directions of the poloidal field and the current. An example of such current is the one produced by ripples in toroidal fields due to the non-ideal coil system of a tokamak [18]. Other sources of torque exist, as for example the torque driven by radio frequency heating [19].

2.2.3 Transport and rotation sinks

Once momentum is deposited in the plasma, inducing local rotation, it is transported radially. Examining the single fluid force balance equation 2.17, transport terms can be identified as being the inertia, pressure gradient and viscosity terms, respectively \( \left( \vec{v} \cdot \nabla \right) \vec{v}, - \nabla p \) and \( - \nabla \cdot \tau \). The origin of these terms is revealed by the procedure of obtaining the first moment of the ion and electron kinetic equations, one of the most elegant derivations of which can be found in [20]. The single fluid stress tensor can be written:

\[
\Pi_{jk} = \Pi_{i,jk} + \Pi_{e,jk} \\
= m_i \int_{f_i} v_j v_k f_i d\vec{v} + m_e \int_{f_e} v_j v_k f_e d\vec{v}
\]
For each position in velocity space, the velocity can be decomposed in the local single fluid velocity and a deviation to it (called particular velocity) \( \vec{v} = \vec{\nu} + \vec{\tilde{v}} \):

\[
\Pi_{jk} = (m_i + m_e) n \nu_j \nu_k + p \delta_{jk} + \pi_{jk} \tag{2.67}
\]

with:

\[
p = m_i \int_{f_i} \vec{\nu}^2 f_i d\vec{\nu} + m_e \int_{f_e} \vec{\nu}^2 f_e d\vec{\nu} \tag{2.68}
\]

\[
\pi_{jk} = nm_i \int_{f_i} \vec{\tilde{v}}_j \vec{\tilde{v}}_k f_i d\vec{\nu} + m_e \int_{f_e} \vec{\tilde{v}}_j \vec{\tilde{v}}_k f_e d\vec{\nu} - p \delta_{jk} \tag{2.69}
\]

Once the divergence of equation 2.67 is taken and the continuity equation 2.16 is used, the first term give rise to inertia, the other two naturally yielding the pressure and viscosity. This expresses the fact that inertia has its origin in spatial disparities of the fluid velocity whereas pressure and viscosity are essentially collective effects originating in the deviation of each particle’s velocity to the local fluid velocity (equations 2.68 and 2.69). These collective effects are governed by the Coulomb collisions between the particles of the plasma.

Based on these interactions between particles which underlie transport, it is possible to obtain another intuitive approach of transport. The particles are subject to a high number of Coulomb interactions, and therefore undergo a random walk during which each collision displaces them to one of the adjacent flux surfaces. The characteristic step of this process is the Larmor radius of the species, while the characteristic time is the collision time on the other species. The random walk results in a diffusion of coefficient for the single fluid \( \rho_{L,i}^2 / \tau_{ei}^{-1} = \rho_{L,e}^2 / \tau_{ie}^{-1} \). This gives a reason why the angular momentum flux due to transport in equation 2.49 is represented in a diffusive form. This simple picture of transport, called classical transport, assumes a straight magnetic field and is known as classical transport. If however the toroidal nature of the magnetic field is taken into account, the step size of the random walk changes considerably. In particular, there appear trapped particles for which the orbit departs from the flux surface by a distance greater than the Larmor radius and which have another characteristic time, their bounce frequency. The transport is therefore increased by an amount depending on the dominant processes (Larmor gyration or orbit precession, collision frequency on the other species or bounce time). This transport is called neoclassical transport [21].

Unfortunately, the levels of transport observed in tokamaks exceed neoclassical predictions. The extra, anomalous, transport, is often attributed to turbulence. Like in any fluid, when the plasma reaches a certain level of motion and thermal agitation, the flow becomes turbulent. The physical quantities of the fluid are subject to fast timescale and short spatial scale fluctuations. One of the sources for these fluctuations is the presence of strong gradient combined with random advection of the attached quantity [22]. The turbulent transport is then produced
by correlation of the fluctuating quantities. It is important to note, that unlike before, where the departure of the particle velocity to that of the fluid were considered, the focus is on the fluctuations in time and space of the fluid quantities themselves. It is possible to decompose the fluid quantity $X$ into its ensemble average (be it over space or time) $\overline{X}$ and its fluctuating part $\hat{X}$ to get $X = \overline{X} + \hat{X}$. Of course, the average of the fluctuating part vanishes, $\overline{\hat{X}} = 0$. In contrast, $\overline{X_1X_2}$ is non-zero if $\hat{X}_1$ and $\hat{X}_2$ are correlated, which is the origin of turbulent fluxes.

For simplicity the example of a turbulent particle flux is explained here, keeping in mind that a similar process occurs for turbulent momentum transport. Considering the single fluid continuity equation (2.16) and decomposing the density and velocity as described above gives:

$$\frac{\partial \overline{n} + \hat{n}}{\partial t} + \nabla \cdot \left( (\overline{\rho} + \hat{\rho}) (\overline{\vec{v}} + \hat{\vec{v}}) \right) = 0$$

(2.70)

This equation accounts for both the slow evolution of the density and its fluctuations. In order to focus on the former exclusively, the ensemble average of equation 2.70 is considered, thus giving:

$$\frac{\partial \overline{n}}{\partial t} + \nabla \cdot \overline{n\vec{v}} + \nabla \cdot \overline{\hat{n}\hat{\vec{v}}} = 0$$

(2.71)

where the additional flux can be identified as arising from correlations in fluctuating quantities, $\overline{\hat{n}\hat{\vec{v}}}$. In the case of turbulent angular momentum transport, the flux is driven by correlations in fluctuations of different components of the velocity.

All the phenomena described above transport the momentum from the core of the plasma towards the edge, but do not remove it from the plasma. Momentum at the edge of the plasma is lost by other processes. The plasma is not a closed system, on the one hand it is constantly fuelled by gas injection at the edge and NBI in the core, on the other hand, it loses particles which cross the last closed flux surface. The momentum of the latter particles is lost from the plasma. An additional momentum sink is due to the friction on the neutrals located at the edge. In this colder region of the plasma, low momentum neutrals arising from gas fuelling are present. The rotating ions undergo charge-exchange reactions on the slow neutrals, replacing a particle carrying momentum by one which has almost none, effectively damping the rotation. It should also be noted that toroidal momentum can be lost by the plasma in the presence of MHD modes, as described in section 2.3.1 and chapter 5.
2.3 Relevance of rotation

2.3.1 Stabilisation of macro-instabilities (MHD)

MHD instabilities, introduced in section 2.1.4, have deleterious consequences on tokamak plasmas, leading to confinement degradation or the termination of the plasma. Overall plasma rotation as well as shear in the rotation profiles have been widely reported to stabilise MHD instabilities, thus greatly enhancing the behaviour of tokamak plasmas.

Among all the stabilisation mechanisms arising from plasma flows, the gyroscopic stabilisation of so-called internal kink mode [23] best illustrates the beneficial role of rotation. The internal kink mode of poloidal and toroidal periodicities \((m, n) = (1, 1)\) consists of a rigid body displacement of a flux tube, the association of a tilt around an axis lying in the equatorial plane and a shift along this same axis. Reference [23] provides a powerful simple analogy with a rotating heavy top being disturbed from its vertical stationary state: a top with sufficient rotation will not fall over but precess around the vertical axis. For a flux-tube-shaped axisymmetric top, it is easy to identify the principal axes of rotation which are the vertical axis, and two perpendicular axes of the equatorial plane (figure 2.4). The moments of inertia along these axes are respectively denoted \((J_\parallel, J_\perp, J_\perp)\), the angular frequency and the height of the top are \(\omega\) and \(h\). Using Euler’s equations from solid dynamics and the conservation of momentum gives the precession rate
of the disturbed top:

\[ \omega_p = \frac{J_\| - J_\perp}{J_\perp} \omega \]  

(2.72)

Applying a displacement \( \vec{\xi} \) to the top's centre of mass along the second principal axis results in an instantaneous rotation around this axis at a rate \( \dot{\xi}/h \), an instantaneous rotation around the third principle axis arising from the precession at a rate \( \omega_p \xi/h \), and a vertical downwards motion of the center of mass by \( \xi^2/(2h) \) (figure 2.4). The energy of the perturbation is therefore:

\[ E_{\dot{\xi},\xi} = \frac{1}{2h^2} \left( J_\perp \dot{\xi}^2 + \frac{(J_\| - J_\perp)^2}{J_\perp} \xi^2 \omega^2 - mgh\xi^2 \right) \]  

(2.73)

Thus, a sufficient condition for stability is:

\[ \frac{(J_\| - J_\perp)^2}{J_\perp} \omega^2 - mgh > 0 \]  

(2.74)

\[ \omega^2 > \frac{J_\perp}{(J_\| - J_\perp)^2} mgh \]  

(2.75)

Here, the instability is driven by gravity, and dimensional analysis enables to define an associated growth rate, \( \gamma = \sqrt{mgh/J_\perp} \) such that the sufficient condition for stability becomes \( \omega > \gamma J_\perp / (J_\| - J_\perp) \). In the case of the displaced flux tube of a tokamak plasma, \( J_\| \sim 2J_\perp \) and the growth rate is given by \( \gamma \sim \epsilon^2 \omega_A \) [24], where \( \omega_A = B_{\text{axis}} / (R_0 \sqrt{\mu_0 \rho}) \) is the Alfvén frequency with \( B_{\text{axis}} \) the magnetic field on axis. The analogy therefore indicates that flows one to two orders of magnitude lower than the Alfvén frequency (\( \omega > \epsilon^2 \omega_A \) with \( \epsilon \sim [0.3, 0.6] \)) can possibly lead to the stabilisation of the \((m,n) = (1,1)\) internal kink mode. The full calculation of the gyroscopic stabilisation is developed in [23], and further stabilising effects of strong rotation are studied in [25].

Another MHD instability avoided in the presence of strong toroidal rotation is the Resistive Wall Mode (RWM) [26]. Scenarios developed with the aim of reaching tokamak steady state operation (appendix B) exhibit a non-monotonic \( q \) profile, due to a broad current profile, together with a peaked pressure profile. In these conditions, the plasma becomes MHD-unstable when the ratio of kinetic to magnetic pressure, \( \beta = 2 \langle p \rangle_V / \mu_0 B_0^2 \) exceeds a threshold called the no-wall limit (the V-subscripted brackets denote a volume average). The shape of the whole plasma then becomes kinked, an instability which would be avoided by fitting an ideally-conducting wall around the plasma. A real wall, however, has a finite conductivity and only reduces the growth rate of the instability to the inverse resistive timescale of the wall \( \tau_w^{-1} = (\mu_0 r_w \Delta_w \sigma_w / 2)^{-1} \), thus giving rise to the RWM. Here, \( r_w, \Delta_w \) and \( \sigma_w \) are the wall’s minor radius position, thickness and conductivity.
Two factors determine $\omega_{MHD}$, the relative rotation of the mode as respect to the wall: the drag force exerted by the latter and the advection of the magnetic structure by the plasma flow. Strong toroidal flows in the plasma will induce a high mode rotation respective to the wall, thus limiting the wall-mode interaction to a superficial region of the wall, of thickness $\delta_w = \sqrt{\frac{2}{\mu_0 \omega_{MHD} \sigma_w}}$. The inverse dependence of the wall skin depth, $\delta_w$, on $\omega_{MHD}$ means that sustaining plasma rotation high above the wall’s inverse resistive timescale renders it effectively ideal and therefore stabilises the RWM. This rotational stabilisation has been predicted, and rotation thresholds calculated [27–30]. Operation above the no wall-limit sustained by strong flows was reported on many tokamaks [31–33]. It is also worth noting that the stabilisation of the RWM can also be achieved using external real time actuators which corrects the RWM induced magnetic perturbation [34] or the interaction with energetic particles [35].

The wall drag mechanism is in itself interesting, as it features a self-feeding interaction between MHD and plasma rotation known as mode-locking [36]. The time-varying magnetic perturbation arising from an MHD instability creates electric fields hence drives eddy currents at the wall’s surface, which in turn generate a magnetic field. This results in a $\vec{j} \times \vec{B}$ force exerted by the plasma on the wall, the opposite force being exerted by the wall on the plasma, leading to a slowing down of plasma rotation. As a result, the skin depth grows larger, meaning that the eddy currents and the magnetic field can penetrate deeper into the wall, braking the plasma even further. If in addition, the MHD mode is further destabilised by the rotation slowing down, its growth is a self-feeding mechanism highly detrimental to the plasma. Several other self-amplifying interactions between MHD instabilities and rotation have been observed, as for example the penetration of error-fields, which are non-axisymmetries in the tokamak magnetic field resulting from the finite number of coils [37], or in the triggering process for Edge Localised Modes (ELM) [38].

Many other MHD modes have been predicted to be stabilised by strong toroidal flows or plasma rotation shear, including the so-called ballooning modes [39–43], ELMs [42,44–46], quasi-interchange modes [47], Mercier modes [48,49] and infernal modes [50].

### 2.3.2 Stabilisation of micro-instabilities (turbulence)

Turbulence arises from fluctuations on short time and spatial scales also known as micro-instabilities. It is thought to be a major drive for transport and thus governs confinement in tokamaks, as presented in section 2.2.3. Turbulent fluids can be considered as composed of eddies of different sizes interacting with each other [22]. To illustrate this, it is useful to use the example of the turbulent flux of particles used in section 2.2.3, $\Gamma_{turb} = \langle n \vec{v} \rangle$. Here, considering a volume containing
an assembly of fluid cells, the fluid density and velocity are taken to fluctuate in space, which can be represented by a Fourier integral:

$$X(\vec{r}) = \int_{\vec{k}} \hat{X}_{\vec{k}} e^{i\vec{k}.\vec{r}} d\vec{k}$$  \hspace{1cm} (2.76)$$

The ensemble averaging of a quantity $Y$ then takes the form:

$$Y = \frac{\int Y(\vec{r}) d^3\vec{r}}{\int d^3\vec{r}}$$  \hspace{1cm} (2.77)$$

The turbulent flux can therefore be written:

$$\vec{\Gamma}_{turb} = \int_{\vec{k}} \hat{n}_{\vec{k}} e^{i\vec{k}.\vec{r}} d\vec{k} \int_{\vec{k}'} \hat{\vec{v}}_{\vec{k}'} e^{i\vec{k}'.\vec{r}} d\vec{k}'$$

$$= \int_{\vec{k}} \int_{\vec{k}'} \hat{n}_{\vec{k}} \hat{\vec{v}}_{\vec{k}'} e^{i(\vec{k} + \vec{k}').\vec{r}} d\vec{k} d\vec{k}'$$

$$= \int_{\vec{k}} \int_{\vec{k}'} \hat{n}_{\vec{k}} \hat{\vec{v}}_{\vec{k}'} \delta(\vec{k} + \vec{k}') d\vec{k} d\vec{k}'$$

$$= \int_{\vec{k}} \hat{n}_{\vec{k}} \hat{\vec{v}}_{-\vec{k}} d\vec{k}$$  \hspace{1cm} (2.82)$$

Equation 2.82 describes an assembly of eddies $\hat{\vec{v}}_{-\vec{k}}$ of size $|\vec{k}|^{-1}$, each transporting a blob of plasma $\hat{n}_{\vec{k}}$. Turbulent transport can here be considered as the random walk of these blobs under the actions of the eddies, the step length being the eddy’s size and the characteristic time the eddy’s lifetime. This random walk picture also means that turbulence transport is likely to be adequately represented by a diffusive mechanism, as mentioned in section 2.2.3. The way these eddies interact with each other can be described by Kolmogorov’s theory \cite{kolmogorov1941}, which applies to general fluids. Examining the Navier-Stokes equation, which is equation 2.17 without the Lorentz force nor external forces:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla) \vec{v} = -\frac{1}{\rho} \nabla p - \mu \nabla^2 \vec{v}$$  \hspace{1cm} (2.83)$$

enables the separation of two contributions to the flow: advection, $(\vec{v}.\nabla)\vec{v}$ and dissipation, $\mu \nabla^2 \vec{v}$. Now considering an eddy of size $l$ and velocity $\hat{v}$, dimensional analysis gives the timescale of the former process, $\tau_{edd}$ = $l\hat{v}^{-1}$, and that of the latter, $\tau_{d} = l^2 \mu^{-1}$. The ordering of these timescale respective to $l$ implies that large eddies essentially disappear while transferring their energy to other eddies by advection, whereas small ones disappear by dissipating their energy. Kolmogorov theory assumes that there is no particular length scale at which energy accumu-
Figure 2.5: Turbulent eddies distortion by sheared flow. Turbulent eddies are stretched by the sheared flow, thus accelerating their advection by other eddies, breaking them into smaller eddies. This process decreases the intensity of the turbulence and reduces the turbulence-driven transport.

lates: energy is injected at the (large) system’s length scale, and the flow adjusts itself to generate an energy cascade from larger to smaller scales until reaching a scale small enough where energy is dissipated. The turbulent energy of each eddy, hence the amplitude of the subsequent fluctuations, depends on the input of energy at the system’s length scale (the drive of turbulence) and the intensity of this cascade transporting the energy to the scale where it is dissipated. For scales larger than this, the lifetime of an eddy is therefore the timescale of the advection process, $\tau_{\text{eddy}}$, a time after which it disappears due to advection by other eddies.

Flow shear can reduce turbulence by modifying the eddy’s lifetime. Considering a bidimensional flow, oriented along the x-direction and sheared in the y direction (figure 2.5), the shear stretches an eddy of size $l$ in the x-direction. After a time $dt$ the eddy’s size in this direction is $|\vec{\nabla}v_x|l dt$. When the eddy is stretched sufficiently, it becomes advected by the neighboring eddies, hence transfers its energy and disappears. In effect, this can be seen as the breaking of the eddy into two smaller eddies. The length at which this occurs is called the correlation length, $l_{\text{corr}}$, and is of order of the eddy’s size. The destruction of the eddy by such a process appears after a time $\tau_{\text{shear}} = l_{\text{corr}}(|\vec{\nabla}v_x|l)^{-1}$. For high flow shear, it is shorter than the timescale of the advection process: the eddy’s lifetime is reduced to $\tau_{\text{shear}}$.

The effect of high flow shear is thus twofold. Firstly, the energy cascade from scale to scale becomes more intense and the turbulent energy dissipated more rapidly. A new balance between the turbulence’s drive and the transport/dissipation process is then reached at lower turbulent energy, meaning a lower eddy velocity. Secondly, due to the breaking of the eddy into two smaller ones as it loses coherence, the scale of eddies in the shear direction is reduced. The
diffusivity of the random walk mentioned earlier in this section is the product of its step size and characteristic velocity, namely the eddy scale in the shear direction and the eddy velocity. These are both decreased by the presence of a high flow shear, the turbulent transport is therefore reduced in the shear direction.

2.4 Summary

In this chapter different formalisms used to represent a plasma are introduced, in particular the single fluid approach relevant to plasma equilibrium stability, and the two fluids description, with the ion fluid relevant to plasma rotation studies. The single fluid and the ion fluid velocities can be considered equal due to the negligibility of the electron mass compared to that of the ion, an assumption which is made throughout this work. It was shown that the transport approach was best suited to rotation studies. The angular momentum is deposited in the plasma by torque sources, the main one being NBI heating. It is then transported and lost at the edge of the plasma, a phenomenon which can be accounted for by a diffusive momentum flux, which is in line with its interpretation as a random walk process. It was also seen that recent theories represent angular momentum transport as the combination of diffusion with a flux resulting from an inward momentum pinch. Lastly, plasma rotation can enhance plasma confinement by helping stabilise MHD instabilities, which was illustrated with the example of gyroscopic stabilisation and rotational stabilisation of the RWM. This RWM case also features the interaction of rotation with MHD modes in a self-amplifying mechanism, whereby the destabilisation of a mode causes a degradation of rotation, which in turn further excites the mode. Other examples of such a deleterious self-feeding interaction can occur in tokamaks, and one of them is considered in chapter 5. Lastly, sheared plasma flows can also reduce turbulent transport by reducing the characteristic size of turbulent eddies.
Chapter 3

Charge eXchange Recombination Spectroscopy

As seen in chapter 2, studying tokamak plasmas requires measurements of different profiles such as density, temperature or magnetic quantities. Momentum transport analysis in particular, is based on time resolved profiles of rotation. This chapter explains the principle of Doppler velocimetry used to obtain the fluid velocity and temperature of a plasma. Its application to tokamak plasmas heated by NBI is called Charge eXchange Recombination Spectroscopy (CXRS). It yields the carbon impurity velocity and temperature profiles, and the conditions in which these can be assumed equal to the main ion fluid are detailed in this chapter. The characteristics of MAST and JET CXRS systems are then presented.

3.1 Doppler velocimetry

A non-invasive way of measuring ion fluid velocity is Doppler velocimetry, a technique on which is based CXRS. The light emitted by an ion travelling at a non-relativistic velocity $\vec{v}_i$ is received by an observer at rest with a Doppler shift in wavelength, $\lambda_i \frac{v_i}{c}$. As seen in chapter 2, the velocity of plasma ions is distributed, meaning that they emit a spectrum of wavelengths. Assuming that each ion to emit a single wavelength in its rest frame, and that the ion distribution function is Maxwellian, as it should be if in equilibrium, then the radiation spectrum also obeys a Maxwellian distribution:

$$P(\lambda) = \sqrt{\frac{m c^2}{2 \pi k T_i \lambda_0^2}} \ e^{-\frac{m c^2 (\lambda_i + \delta \lambda - \lambda_0)^2}{2 k T_i \lambda_0^2}}$$  \hspace{1cm} (3.1)

Here, $\delta \lambda$ is the shift between the emission frequency of the ion at rest ($\lambda_i$) and the central value of the Maxwellian. The ion fluid velocity is then given by $v_i = c \frac{\delta \lambda}{\lambda_i}$. The full width half maximum (FWHM) of the distribution, $\Delta \lambda$ is a measure of the
CHAPTER 3. CHARGE EXCHANGE RECOMBINATION SPECTROSCOPY

Figure 3.1: The departure $\delta \lambda$ of the central wavelength of the Maxwellian spectrum, $\lambda_0$ to the wavelength of the emitting transition, $\lambda_i$ gives the ion velocity. The FWHM, $\Delta \lambda$ yields the ion temperature. The spectrum given here is a typical CXRS spectrum from a MAST plasma.

3.2 Charge exchange and carbon spectroscopy

3.2.1 Principle

An application of Doppler velocimetry is Charge eXchange Recombination Spectroscopy (CXRS, [52]). A plasma ion can emit at a single wavelength when an electron bound to it undergoes a transition between two quantum states. This is called line-emission. The fully ionised deuterium plasma does not give rise to such line-emission. Deuterium line-emitting neutrals are however present in the colder edge of the plasma, and along the trajectory of the NBI beam. The collision between these neutrals and fully ionised ions can lead to a charge exchange reaction and the production of a partially ionised ion in an excited state. This ion then relaxes to a lower energy state by line-emission. In the case of impurity carbon, this reaction is:

$$C^{6+} + D \rightarrow C^{5+*} + D^+ \quad (3.2)$$
$$C^{5+*} \rightarrow C^{5+} + h\nu \quad (3.3)$$

CXRS uses the line emission from carbon impurity ions, at a wavelength of 529nm. This corresponds to a transition from the $n = 8$ to the $n = 7$ quantum energies, and is in the visible region, meaning that a wide range of spectroscopy techniques are available for the analysis. Carbon is a component of the first wall and traces of
it are naturally present in tokamak plasmas. Additionally, it emits in a wavelength region which is somewhat simple to distinguish from that of the deuterium neutrals.

The measurements by CXRS can be considered local because the light collected by a chord of the spectroscopic system mainly originates from a small volume, at the intersection of the neutral beam path and the chord’s line of sight, where the charge exchange reaction occurs with the high energy beam neutrals. This is referred to as “active emission” (figure 3.2).

The spectrometer however collects light from other sources than this active CX emission, which is called background light and has to be removed from the analysed spectra. Some parasitic emission occurs due to charge exchange reactions at the intersection of the line of sight and the plasma edge, which is called “passive” (parasitic) emission. There can also be an active parasitic emission if the line of sight crosses the trajectory of a second neutral beam (figure 3.2). Other line-emission from partially ionised impurities or resulting from other charge exchange reactions than the deuterium-carbon recombination can also pollute the spectrum. A continuum parasitic emission is generated by bremsstrahlung, due to the acceleration of the charged particles in the plasma. To substract the background light from the measured spectra, one possibility is measuring it while the neutral beam is off or when it is not in the path of the line of sight. Alternatively, the background emission can be modelled based on the plasma impurity content and the characteristics of the edge.

Figure 3.2: The CXRS diagnostic in MAST and the origin of the active emission of interest and the parasitic emissions composing the background light. For simplicity, only one of the 64 lines of sight is shown.
CHAPTER 3. CHARGE EXCHANGE RECOMBINATION SPECTROSCOPY

Figure 3.3: The snapshot analysis gives the ion temperature and velocity profiles at the beam shut off time. The background emission is estimated during the 5ms following beam shut off. The plasma evolution, occurring on a confinement timescale ($\sim 50$ms), is negligible during this time.

3.2.2 Limitations

Some additional phenomena limit the performance of the CXRS diagnostic. Even if the velocity distribution function of the carbon impurity is Maxwellian, the charge exchange cross-section depends on the velocity of the carbon ion relative to that of the beam neutral. This means that a certain part of the carbon velocity distribution function is more prone to charge exchange reactions, resulting in distorted carbon spectra hence error in the measurements [53]. Since the neutral beam energy is of order 50keV to 100keV in present day tokamaks, this does not affect MAST with lower temperature plasmas where $T_i \sim 1$keV, the dispersion in the carbon velocity distribution being negligible in this case. In JET high temperature discharges however, where $T_i \sim 10$keV, this cross section effect has to be accounted for in order to yield accurate CXRS data.

The natural line width of the observed carbon transition is negligible compared to the transition wavelength. Nevertheless, taking into account the spin of the electron means that the eight $n=8$ and seven $n=7$ levels are not degenerate anymore due to spin orbit interaction. Because of this fine structure, the average width of the transition is finite and corresponds to a 4eV temperature. The presence of the magnetic field splits the energy level further by Zeeman effect, broadening the width of the transition to a temperature equivalent of order 100eV.

It is important to note that the temperature and velocity measured are those of the carbon impurity, and not those of the deuterium fluid. Energy transfers between the ion and impurity fluid occur on faster timescales than the energy confinement time, enabling to confidently make the assumption that their temperature are equal. Neoclassical calculations enable to estimate the departure of the impurity fluid velocity from that of the ion fluid [54]. It was found that this departure was significant where the second radial derivative of the temperature is large [55].
3.3 CXRS at MAST

MAST is heated by two neutral beam injectors, denoted south-west (SW) and south (SS). A set of 64 lines of sight for CXRS observes each beam (figure 2.1), which corresponds to a spatial resolution $\sim 1\text{cm}$, while the time resolution is 5ms. Because of the small size of MAST plasmas and their relatively low temperature, some neutrals can penetrate until plasma mid-radius. The density of the neutrals surrounding the plasma is also relatively high. This leads to high levels of background emission, making the analysis of the CXRS data technically difficult. The collected spectra are analysed by two different methods, corresponding to two ways of subtracting the background light from the raw data.

The first method yields a single profile at the time when one of the beams switches off, $t_{\text{off}}$, and is commonly referred to as the snapshot analysis. The background emission of the plasma is given by the spectrum integrated between $t_{\text{off}}$ and $t_{\text{off}} + 5\text{ms}$. It is substracted from the spectrum integrated between $t_{\text{off}} - 5\text{ms}$ and $t_{\text{off}}$, which includes the active and background signal. This background subtraction is possible because the ion velocity and temperature evolve on a timescale given by the momentum and energy confinement times, of order 50ms, which are long compared to the integration time. The obtained spectrum is fitted to give the ion velocity and temperature at $t_{\text{off}}$ (figure 3.3).

The second method, known as the time-resolved analysis, outputs time-resolved profiles for the whole plasma discharge, looking at the SW and the SS beams at the same time. It relies on the assumption that the SS and SW chords observe
the same background light, due to the axisymmetry of its source. When the shot is heated by a single beam, the set of chords looking at the heating beam is taken as the active set of chords, while the other set measures the background emission of the plasma. Nevertheless, because both systems are made of different optics and fibers, they have different transmittivities: for the same number of photons emitted by the plasma, they do not measure the same signal amplitude. This is compensated using a scalar correction factor relying on the continuum emission of the plasma before carrying out the background subtraction. The fitting of the active spectrum then yields the ion velocity and temperature. Profiles calculated using this method are shown in figure 3.5.

Until 2008, MAST was fitted with two different beams with different acceleration energies and penetrations hence producing an active signal with same central wavelength and FWHM, but different intensity. After transmittivity difference correction, the background emission measured on the SW and SS chord is identical, and the difference of these is a Maxwellian spectrum yielding the ion velocity and temperature. This procedure is shown on figure 3.4.

Following an upgrade in 2008, both MAST neutral beam injectors became identical, which ruled out the above procedure for the time-resolved analysis, because the active signal amplitude on the SW and SS chords became identical. As a consequence, a third set of lines of sight was installed to measure the background light, which after transmittivity difference correction, is subtracted from the light measured on the beams. This is effectively the same procedure as the pre-2008 one beam time-resolved protocol described before.
3.4 CXRS at JET

The JET core CXRS system has a spatial resolution $\sim 8\text{cm}$ and a time resolution of $10\text{ms}$. JET plasmas are hotter and larger than MAST ones, so that the observed spectrum is dominated by the active charge exchange signal. As a result, raw data from the CXRS system can be modelled taking into account the active emission and the background light due to other parasitic transitions (figure 3.6). This allows the identification of the charge exchange active signal and deduction the ion velocity and temperature by Doppler velocimetry. The cross-section effect described in section 3.2.2 are corrected for and the deuterium fluid velocity can be modelled, although not routinely, from that of the carbon fluid. Profiles measured using CXRS on JET are shown in figure 3.7.
3.5 Summary

This chapter introduced the main diagnostic used to measure the ion velocity and temperature profiles in NBI-heated tokamak plasmas, charge exchange recombination spectroscopy. It relies on the analysis of the spectrum of Doppler shifted carbon line-emission, and the carbon impurity velocity and temperature can be considered equal to those of the ion fluid under certain assumptions. CXRS is implemented on the MAST tokamak, with a resolution of $\sim 5$ms and $\sim 1$cm, and on JET with a resolution of $\sim 10$ms and $\sim 8$cm.
Chapter 4

Database study of plasma rotation at JET

This chapter investigates the broad trends of plasma rotation in order to identify its general magnitude in JET and the parameters that influence it. In addition, the work described here aims at identifying dependences over large numbers of discharges that can be linked to theoretical predictions about confinement in tokamak plasmas. This is attempted by means of the statistical analysis of a large purpose-built database. This chapter first introduces the theoretical justifications of such statistical studies, namely the dimensional analysis and scale invariance theories, and gives an overview of the existing H-mode international database. It then presents the rotation database built at JET, with details concerning its construction, content and the assessment of its quality. The database enabled the study of the rotation of JET plasmas based on the thermal and Alfvén Mach numbers, the main trends of which are discussed. The dependence of rotation on plasma parameters was investigated in more detail by deriving scalings of the Mach numbers. Since rotation is not only determined by its source, but also by angular momentum confinement, the scaling of the momentum confinement time was also studied. This was extended to the energy confinement time, in order to unveil a possible confinement enhancement with rotation. Due to the strong theoretical background provided by dimensional analysis and scale invariance, the dimensional scalings of confinement times were converted into a dimensionless form.

4.1 Relevance of database studies

4.1.1 Theory

Database studies can provide significant help to investigate physical problems for which the theory is not completely known or highly complex. It allows the identification of broad trends and the derivation of empirical laws, useful for the design or
upgrade of experimental devices. Furthermore, it can provide guidance on how a system will behave in a region of parameter space which has not yet been explored. Last but not least, it potentially provides with statistical evidence of theoretical predictions and can give indications of which physical models are applicable to tokamak plasmas.

Several simple, yet powerful, methods have been developed with this aim, relying on basic physical concepts. The first one is dimensional analysis, which exploits the fact that the behaviour of a system does not depend on the physical units that are used to describe it. Considering a variable of interest \( Y \) assumed to depend on \( N \) parameters, \( Y = \mathcal{F}(X_1, X_2, \ldots X_N) \). Each of these quantities has a unit depending on fundamental physical dimensions (length, time, weight, mass...). The number of fundamental dimensions needed to express all the units involved is \( k \), with \( k \leq N + 1 \). Thus, there exist \( k \) parameters, the units of which can be used to express the units of the \( N + 1 - k \) other variables in the form of power products.

Reordering the \( X_i \), \( i \in \{1, \ldots N\} \) such that these \( k \) parameters are \((X_1, X_2, \ldots X_k)\), all other variables can be made dimensionless by the transformation:

\[
\Pi_i = \frac{Y}{X_1^{\alpha_{0,1}} X_2^{\alpha_{0,2}} \ldots X_k^{\alpha_{0,k}}} \quad (4.1)
\]

\[
\Pi_1 = \frac{X_{k+1}}{X_1^{\alpha_{1,1}} X_2^{\alpha_{1,2}} \ldots X_k^{\alpha_{1,k}}} \quad (4.2)
\]

\[
\ldots 
\]

\[
\Pi_{N-k} = \frac{X_N}{X_1^{\alpha_{N-k,1}} X_2^{\alpha_{N-k,2}} \ldots X_k^{\alpha_{N-k,k}}} \quad (4.4)
\]

The dependence of the \( Y \) variable can then be written in the form:

\[
\Pi = \mathcal{F}_\Pi (X_1, X_2, \ldots X_k, \Pi_1, \Pi_2, \ldots \Pi_{N-k}) \quad (4.5)
\]

with, \( \mathcal{F}_\Pi = X_1^{\alpha_{0,1}} X_2^{\alpha_{0,2}} \ldots X_k^{\alpha_{0,k}} \mathcal{F} \), an unknown function which can be arbitrarily intricate, but is dimensionless. \( \mathcal{F}_\Pi \) cannot depend on the units used to formalise the problem, hence must be left unchanged by any units changes. This implies that \( \mathcal{F}_\Pi \) does not depend on the dimensional variables \((X_1, X_2, \ldots X_k)\):

\[
\Pi = \mathcal{F}_\Pi (\Pi_1, \Pi_2, \ldots \Pi_{N-k}) \quad (4.6)
\]

This is the Vaschy-Buckingham theorem [56, 57]. Enlightened physical intuition and knowledge is of course required in the choice of the parameters \((X_1, X_2, \ldots X_N)\) to be included while addressing the problem. Furthermore, the solution is not complete, as only the dependence of \( Y \) is determined, the \( \mathcal{F}_\Pi \) function remaining unknown. Theory by Kadomtsev [58, 59] identified \( \{a, R_0, n, B_\phi, 0, B_p, m_e, m_i, T_e, T_i, c, \epsilon_0, e\} \) as a set of quantities describing a tokamak plasma. Here, profile
quantity like $n$, $T_e$, $T_i$, $B_p$ can be taken as line averaged quantities, and $c$ is the speed of light in vacuum. In this formulation, $N = 12$, and all dimensions can be expressed as a function of \{$a, c, e, m_i$\}, meaning $k = 4$. The dimensionless ratios $\Pi_i$ can be defined as \{$R_0/a, B_p/B_\phi, m_e/m_i, T_e/T_i, eB_0a/(m_ic), T_i/(m_ic^2)$, $na^3, ac^2m_ie_0/e^2$\}. Further modifications can yield a somewhat more usual set of dimensionless parameters, \{$\epsilon_p, q_{95}, m_e/m_i, T_e/T_i, \rho^*, v_{th,i}/c, na^3, \delta_s/a$\} where $\rho^*$ is the ion Larmor radius normalised to the plasma minor radius, $q_{95} = q_{L,i}/a$, and $v_{th,i}$ is the ion thermal speed. The safety factor at $\psi_N = 0.95$ and the plasma skin depth. Then according to the Vaschy-Buckingham theorem, the energy confinement time dependence should be 
\[
\tau_E = \frac{W_{kin}}{P_{in} - \frac{\partial W_{kin}}{\partial t}}
\] (4.7)
with $W_{kin}$ and $P_{in}$ the total plasma thermal energy and auxiliary power.

Another approach consists in analysing the equations describing the system and determining the variable scaling transformations that leave them invariant. This is known as scale invariance. Assuming a dependence of the solution on the variables, this solution has to be left unchanged by the same transformations. This is best seen by examining one of the derivation given in [60, 61]. Considering the kinetic equation 2.5 and assuming the plasma to be collisionless yields the Vlasov equation:
\[
\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla_r f_\alpha + \frac{1}{m_\alpha} \left( q_\alpha \left( \vec{E} + \vec{v} \times \vec{B} \right) + \vec{F}_{ext} \right) \cdot \nabla_v f_\alpha = 0
\] (4.8)
It is possible to scale the variables of this equation:
\[
f_\alpha \rightarrow \mathcal{A}_f f_\alpha, \quad \vec{v} \rightarrow \mathcal{A}_v \vec{v}, \quad \vec{r} \rightarrow \mathcal{A}_r \vec{r}, \quad t \rightarrow \mathcal{A}_t t, \quad \vec{E} \rightarrow \mathcal{A}_E \vec{E}, \quad \vec{B} \rightarrow \mathcal{A}_B \vec{B}
\] (4.9)
where quasi-neutrality guarantees that the ion and electron distribution functions are to be scaled by the same factor $\mathcal{A}_f$. This class of transformations leaves equation 4.8 unchanged if:
\[
\begin{align*}
\mathcal{A}_t &= \mathcal{A}_x \mathcal{A}_v^{-1} \\
\mathcal{A}_E &= \mathcal{A}_v^2 \mathcal{A}_x^{-1} \\
\mathcal{A}_B &= \mathcal{A}_v \mathcal{A}_x^{-1}
\end{align*}
\] (4.10)
This means that equation 4.8 is invariant under three transformations only:

$$
\begin{align*}
&f_\alpha \mapsto A f_\alpha \\
&\vec{v} \mapsto A \vec{v} \\
&t \mapsto A^{-1} t \\
&E \mapsto A^2 \vec{E} \\
&B \mapsto A \vec{B} \\
&r \mapsto A \vec{r} \\
&t \mapsto A t \\
&\vec{E} \mapsto A^{-1} \vec{E} \\
&B \mapsto A^{-1} \vec{B}
\end{align*}
\quad (4.11)
$$

Assuming a power scaling for the energy confinement time $\tau_E \propto n^{\alpha_n} T^{\alpha_T} B^{\alpha_B} \phi$, this expression for $\tau_E$ must also be invariant under the transformations 4.11. This corresponds to three constraints on the exponents $\alpha_n$, $\alpha_T$, $\alpha_B$ and $\alpha_a$:

$$
\begin{align*}
\alpha_n &= 0 \\
2\alpha_T + \alpha_B + 1 &= 0 \\
\alpha_a - \alpha_B - 1 &= 0
\end{align*}
\quad (4.12)
$$

thus yielding a scaling of the type $\tau_E \propto B^{-1} \left( \frac{T}{\nu_{\phi,0} \phi} \right)^{\alpha_B}$ for the energy confinement. This can be generalised by taking a Taylor series instead of a power product for the dependent function, which then yields $\tau_E = B^{-1} \tilde{F} \left( \frac{T}{\nu_{\phi,0} \phi} \right)$. Further normalisation would yield the dimensionless expression $\tau_E = \omega_{L,i}^{-1} \tilde{F}(\rho_*)$. Here again, physical intuition must be used in the choice of the equations representing the behaviour of the system. References [60,61] applies the scale invariance principle for different models including various effect in the plasma’s behaviour. Taking collisions into account in the kinetic equation 2.5, and including Maxwell’s equations 2.1-2.4, yields the dependence $\tau_E = \omega_{L,i}^{-1} \tilde{F}(\rho_*, \nu_*, \beta, q_{95}, \epsilon_p, \kappa)$ with $\kappa$ the elongation of the plasma. $\nu_*$ is the normalised ion collision frequency $\nu_* = \nu_{ii} (\epsilon \omega_{b,i})^{-1}$ with $\nu_{ii}$ the ion-ion collision frequency and $\omega_{b,i}$ the ion thermal bounce frequency $\omega_{b,i} = v_{th,i} \sqrt{2 \epsilon} (R_0 q)^{-1}$. $\epsilon$ is the local aspect ratio.

### 4.1.2 The H-mode confinement database

An international database has been created in order to provide experimental data on which to base the dimensional analysis and scale invariance methods detailed in section 4.1.1 [62-64]. It includes the data of H-mode plasmas (appendix B) in 18 tokamaks world-wide, with different sizes and shapes, aiming at enabling extrapolation of the energy confinement in ITER, and shed light on the physics at stake. This database contains a very large number of entries ($\sim 10^4$) and includes parameters such as density, magnetic field or input power. A regression analysis using ordinary least square fitting yields a scaling of the energy confinement time:

$$
\tau_{E,IPB09(y,2)} = 5.62 \times 10^{-2} I_{p}^{0.95} B_{\phi,0}^{0.15} P_{in}^{-0.69} n^{0.41} R_{0}^{1.97} \epsilon_{p}^{0.58} \kappa^{0.78}
\quad (4.13)
$$

where $I_p$, $B_{\phi,0}$, $P_{in}$, $n$ and $R_0$ are the plasma current, central magnetic field, auxiliary heating power, line average density and plasma major radius. This scaling
gives the empirical dependence of the energy confinement time on engineering parameters that are directly controlled in tokamak experiments. It can be converted into a scaling based on dimensionless parameters [64, 65] (such a conversion is carried out in sections 4.4.3), for which dimensional analysis and scale invariance provide a theoretical background:

\[
\omega \propto \nu^{-0.01} \rho^{-2.70} \beta^{-0.90} q_{95}^{-3.0} \epsilon_{\nu}^{0.73} \kappa^{3.3} \quad (4.14)
\]

In this scaling, \( \beta \) is the plasma’s kinetic energy normalised to that of the magnetic field, \( \beta = \langle nT \rangle_V \left( \frac{1}{2} \mu_0 B_0^2 \right)^{-1} \), where the angled brackets represent a volume average. The validity of scalings 4.13 and 4.14 depend on the model selected leading to the parameters included in them. It also assumes a power law dependence, as a compromise between generality and fitting possibilities. Although not providing an exact prediction of the energy confinement in ITER, it may be considered as giving a rough estimate of it, within the assumptions made. Such scalings can also shed light on which transport mechanisms may be dominant and which theory is most likely to be relevant to fusion plasmas [61, 64–66]. It should be noted that studies of scale invariance in [60, 61] and analyses of energy confinement time scaling in [64] both do not investigate the influence of rotation, despite its potential beneficial role in confinement.

### 4.2 The JET rotation database

#### 4.2.1 Objectives

The JET rotation database was built with the aim of identifying the general magnitude of plasma rotation and characterising the relevant parameter dependences of plasma rotation and momentum confinement. The general trends of confinement in JET plasma are also meant to be linked to theoretical predictions and transport models. Scalings of confinement times are derived and discussed, with an eye towards the identification of a possible enhancement with rotation. This database includes data from a single machine, JET, and therefore does not allow any extrapolation to future devices.

#### 4.2.2 Structure

The database contains entries from various operational scenarios such as the ELMy H-mode baseline scenario, hybrid scenario and steady-state scenario (appendix B). Here, we refer to the steady-state scenario as the ITB scenario, due to its reliance on internal transport barriers (appendix B). The entries were carefully selected to be MHD-stable, meaning that they do not exhibit any macro-scale MHD instability as for example sawteeth, NTMs or RWMs. This implies that turbulence...
is expected to be the dominant transport mechanism for the plasmas included in the database. The dominant auxiliary heating system is NBI, which is also a torque source (section 2.2.2), thus sustaining central rotation as high as $\omega_\phi \sim 220 \text{krad.s}^{-1}$.

To cover a broad range of rotation values, plasmas with dominant ion cyclotron resonance heating (ICRH), and therefore with low torque input, have been included in the database. This also has the benefit of decoupling the auxiliary heating power and the torque input in the dataset. These database entries are reliable in the sense that they have been subject of careful analysis and selection in order to appear in other publications. They are a subset of dedicated databases for each of the operation scenarios included, such as the main JET H-mode confinement database [64] and those for the hybrid [67] and ITB scenarios [68]. The entries with predominant ICRH heating are mainly taken from experiments that studied ICRH-driven plasma rotation [69, 70]. Furthermore, the database contains an additional subset of JET plasmas from experiments with reversed plasma current and toroidal field direction, i.e. counter-current NBI. The database includes 574 entries and the repartition between scenarios is given in table 4.1.

All plasmas are analysed in a steady-state phase, and to reduce measurement statistical errors, each data signal is averaged over a 200ms time-window, a time span which is of the order of the confinement time. The database parameters fall into four categories: general, energy, rotation and profile parameters. The velocity and temperature of the ion fluid is assumed equal to that of the carbon impurity fluid, measured by CXRS. Although these were shown to differ [55], the correction for H-mode discharges never exceeds 5% which is within the accuracy of the diagnostic. A summary of the main parameters for each category is given in table 4.2. Existence diagrams of various parameters are shown in figure 4.1. Besides the ones mentioned in table 4.2, other parameters are included in the database, as for instance normalised gradient lengths of ion temperature and rotation quantities, thermal and momentum diffusivities or parameters characterizing ITB strength.

Table 4.1: Summary of the operation scenarios in the JET rotation database. The table shows the symbols used to represents the different scenarios in subsequent figures. Part of the database has an overlap with other experimental databases as indicated in the last column.

<table>
<thead>
<tr>
<th>Plasma scenario</th>
<th>Entries</th>
<th>Symbol</th>
<th>Connecting database</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELMy H-mode</td>
<td>239 + 60</td>
<td>●●○○</td>
<td>H-mode confinement database [64]</td>
</tr>
<tr>
<td>Counter NBI</td>
<td>37</td>
<td>○</td>
<td>ICRH rotation experiments [69, 70]</td>
</tr>
<tr>
<td>Dominant ICRH</td>
<td>65</td>
<td>□</td>
<td>Hybrid database [67]</td>
</tr>
<tr>
<td>Hybrid</td>
<td>110</td>
<td>△</td>
<td>ITB database [68]</td>
</tr>
<tr>
<td>ITB</td>
<td>63</td>
<td>▲</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>574</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Table 4.1]
CHAPTER 4. DATABASE STUDY OF PLASMA ROTATION AT JET

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Unit</th>
<th>Range</th>
<th>Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron line-integrated density</td>
<td>(n_{el})</td>
<td>(10^{19})m(^{-2})</td>
<td>3.90-28.2</td>
<td>1%</td>
</tr>
<tr>
<td>Central magnetic field</td>
<td>(B_{\phi,0})</td>
<td>T</td>
<td>0.99-3.56</td>
<td>2%</td>
</tr>
<tr>
<td>Plasma current</td>
<td>(I_p)</td>
<td>MA</td>
<td>0.98-3.99</td>
<td>3%</td>
</tr>
<tr>
<td>Total input power</td>
<td>(P_{in})</td>
<td>MW</td>
<td>1.85-31.3</td>
<td>6%</td>
</tr>
<tr>
<td>NBI input power</td>
<td>(P_{NBI})</td>
<td>MW</td>
<td>1.85-21.0</td>
<td>2%</td>
</tr>
<tr>
<td>Total kinetic energy</td>
<td>(W_{kin})</td>
<td>MJ</td>
<td>0.51-9.09</td>
<td>5%</td>
</tr>
<tr>
<td>Energy confinement time</td>
<td>(\tau_E)</td>
<td>s</td>
<td>0.067-0.493</td>
<td>8%</td>
</tr>
<tr>
<td>Central angular frequency</td>
<td>(\omega_0)</td>
<td>krad.s(^{-1})</td>
<td>2.46-222</td>
<td>10%</td>
</tr>
<tr>
<td>Total toroidal angular momentum</td>
<td>(L_\phi)</td>
<td>N.m.s</td>
<td>0.04-9.04</td>
<td>10%</td>
</tr>
<tr>
<td>Toroidal torque</td>
<td>(T_\phi)</td>
<td>N.m</td>
<td>0.31-23.1</td>
<td>2%</td>
</tr>
<tr>
<td>Momentum confinement time</td>
<td>(\tau_\phi)</td>
<td>s</td>
<td>0.049-0.585</td>
<td>8%</td>
</tr>
<tr>
<td>Profile average thermal Mach number</td>
<td>(\langle M_{th}\rangle_p)</td>
<td></td>
<td>0.02-0.62</td>
<td>5%</td>
</tr>
<tr>
<td>Profile average Alfén Mach number</td>
<td>(\langle M_A\rangle_p)</td>
<td></td>
<td>0.0009-0.07</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 4.2: Overview of the main database parameters.

The database is automatically generated from a list of shot numbers and time windows by a routine written especially for this purpose. The emphasis was laid on finding a good compromise between computing time and accuracy of the data. This routine was designed to generate and update the database easily, and the output can be used in any data analysis software. In addition to the actual data for each entry, the database contains all information on data computation procedures, estimated error bars and possible computing errors that have occurred.

4.2.3 Data validation, errors and correlations

Plasmas from the database were performed over a large time span, meaning that the wall conditioning, or even the physical structure of JET, which underwent several divertor modifications, has changed. These features are sources of data scatter and may confuse the analysis. For each parameter of the database, a single relative error indicates the level of uncertainty on the value of all entries, not taking into account individual plasma conditions which might influence data scatter. Table 4.2 gives the error for the main parameters of the database.

The size of the database implies that the accuracy of every single data point cannot be individually guaranteed. Nevertheless all orders of magnitude are checked as well as consistency with basic or well-known scalings, as for instance the correlation of torque and angular momentum (figure 4.1-(a)). For the counter-current data, the absolute value of the torque and rotation is shown, which is done throughout this chapter. The toroidal torque is that supplied by the tangential NBI system of JET and no other sources of momentum have been considered. The data scatter observed in this graph is expected due to differences in momentum confinement.
Figure 4.1: Existence diagrams of several database parameters for various scenarios. (a) The total toroidal angular momentum as a function of the toroidal torque. (b) The central magnetic field as a function of plasma current. (c) The toroidal torque as a function of the auxiliary heating power. (d) The density as a function of the NBI heating power.
Depending on how they were calculated, the database parameters are validated by different means. For parameters characterizing the overall discharge such as line-integrated density, the heating powers or the plasma current, a basic handcheck is done during the time-window determination. An in-depth look at all parameters for every outlier observed while using the database is also taken. As noted before, parameters computed by means of a more complex calculation, for instance by use of a scaling law, are benchmarked by comparison with available results from other computation methods or databases.

In contrast to the scan of a given parameter keeping all other parameters constant, the entire parameter set usually varies from one database entry to another, meaning that the parameter space is not covered methodically. In addition, the database features parameter correlations which are detrimental to the quality of the regression analyses. The correlations between the logarithms of a number of relevant parameters are shown in table 4.3. The logarithm is used here because of the log-linear regression analyses carried out on the data (section 4.3 and appendix C). The magnetic field, $B_{0\phi}$, and plasma current, $I_p$, are not entirely independent, because of operation at distinct values of edge safety factor $q_{95}$. This is also evident from figure 4.1-(b). Similarly, in figure 4.1-(c), a correlation between the auxiliary heating power, $P_{in}$ and the toroidal torque $T_\phi$ is seen. This is because NBI, which also provides the toroidal torque, is the predominant heating system on JET. Indeed, 60% of the database entries, including almost all H-modes, have a fraction of NBI heating higher than 90% as seen in figure 4.2. This figure shows that the inclusion of predominantly ICRH entries (NBI heating fraction lower than 70%) extended the $P_{in}/T_\phi$ range. Hybrid and ITB plasmas have a different heating scheme from that of baseline H-modes, hence further decorrelate $P_{in}$ and $T_\phi$. Nevertheless, even though including several plasma scenarios breaks the coupling between these parameters, it may well confuse the regression analyses by grouping very different confinement modes.

A proper analysis of trends and scaling requires a sufficient range for the involved parameters. The relevant ranges for the database parameters are given

\[
\begin{array}{cccccc}
\ln n_{el} & \ln I_p & \ln B_{0\phi} & \ln P_{in} & \ln T_\phi & \ln M_a \\
+1.000 & +0.423 & +0.116 & +0.437 & +0.493 & +0.426 \\
+1.000 & +0.749 & +0.372 & +0.362 & +0.209 & +0.073 \\
+1.000 & +0.362 & +0.100 & -0.199 & +0.799 & +0.528 \\
+1.000 & +0.799 & +0.528 & +0.817 & +1.000 \\
+1.000 & +0.817 & +1.000 & +1.000 \\
\end{array}
\]

Table 4.3: Correlation matrix for the natural logarithm of some of the parameters in the database.
in table 4.2. The experiments in each scenario are performed using standardized parameters and discharges are often repeated with most of the parameters kept nominally equal. These parameters are therefore distributed in clusters rather than uniformly, which may be detrimental to the derivation of robust scaling laws. One method to analyse the database quality is the so-called principal component analysis (appendix C). This analysis showed that using models with up to four or five parameters in order to fit scaling laws to the database is reasonable.

## 4.3 Rotation of JET plasmas

### 4.3.1 Thermal Mach number

Rotation is most conveniently expressed in terms of dimensionless parameters, which are widely used in theoretical work. They can also make it easier to compare rotation between scenarios or even devices. The thermal Mach number is for instance used in turbulence studies \[22\], it is defined as the ratio of the toroidal fluid velocity and the thermal velocity:

\[
M_{th} = \frac{v_\phi}{v_{th,i}} = v_\phi \sqrt{\frac{m_i}{eT_i}}
\]  

(4.15)

where \( T_i \) is expressed in eV units. The thermal Mach number defined here is a profile value, the JET rotation database requires a scalar and therefore uses the profile averaged thermal Mach number, \( \langle M_{th} \rangle_p \).

The average thermal Mach number in JET main scenarios (H-mode, hybrid and ITB, which are often predominantly NBI-heated) was observed in the range \( 0.2 \leq \langle M_{th} \rangle_p \leq 0.6 \). Considerably lower values are found for predominantly ICRH-heated discharges, which have a torque less than 1N.m, with average thermal
Mach numbers in the range $0.02 \leq \langle M_{th}\rangle_p \leq 0.14$. The scenario mean values are $\langle M_{th}\rangle_p = 0.36 \pm 0.09$ for type I ELMMy H-mode, $\langle M_{th}\rangle_p = 0.25 \pm 0.07$ for type III ELMMy H-mode, $\langle M_{th}\rangle_p = 0.34 \pm 0.06$ for the hybrid scenario and $\langle M_{th}\rangle_p = 0.31 \pm 0.08$ for discharges with ITBs. The values quoted here are the mean of the database subsets and its standard deviation. It should be noted that a significantly lower thermal Mach numbers is found for discharges with type III ELMs, which could be explained by the dependence of ELM induced losses and H-mode pedestal regeneration time on ELM frequency \cite{71, 72}.

The average thermal Mach number is found to scale approximately with the ratio of toroidal torque to the total auxiliary power in figure 4.3-(a). Although these parameters are correlated in JET (figure 4.1-(c), table 4.3), resulting in a restricted range of their ratio, the scaling in figure 4.3-(a) remains significant. This means that the ratio of rotation and square root of the temperature scales with the ratio of their source. Such a dependence can only be observed if there is a certain degree of coupling in energy and momentum confinement, a coupling which is consistent with electrostatic turbulence theory \cite{73}.

The profile shape of the Mach number can also be of importance in tokamak studies. The peaking factor, defined here as the central value divided by the profile average value, can account for the shape of the profile. Figure 4.3-(b) shows the peaking factor of the thermal Mach number $p_{M_{th}}$ as a function of density. If the peaking factor is unity, the rotation profile shape is similar to that of the square root of the temperature. The Mach number profile is more peaked for low density, predominantly NBI-heated ITB and hybrid discharges, with peaking


1.4, 1.0, 0.6, 0.4, 0.2, 0.80

ψ

N

1/2

\( M_{\text{th}} (\psi/N_{\text{th}}) / <M_{\text{th}} > \)

Type I ELMy H-mode (#53298, \( t = 20.0\text{s}, <M_{\text{th}} >= 0.53 \))

Counter NBI (#59630, \( t = 21.8\text{s}, <M_{\text{th}} >= 0.16 \))

ITB Scenario (#52881, \( t = 10.3\text{s}, <M_{\text{th}} >= 0.33 \))

Figure 4.4: Typical profile shape of the thermal Mach number for three different plasma scenarios of the JET database. The profile shape is given by the local value divided by the profile average value.

Figure 4.4: Typical profile shape of the thermal Mach number for three different plasma scenarios of the JET database. The profile shape is given by the local value divided by the profile average value.

factors up to 1.8, while it is almost flat for high-density H-modes. Hollow Mach profiles, meaning \( p_{\text{Mth}} < 1 \), are found for a number of discharges in the database (figure 4.3-(c)). Hollow thermal Mach profiles either mean that the ion temperature profile is strongly peaked while the rotation profile remains flat or that the rotation profile itself is hollow. Some entries with hollow thermal Mach profiles are found to be high density counter-current NBI discharges. These are characterized by flat rotation profiles linked to a torque deposition located more off-axis, while the power deposition peaks on-axis. The majority of hollow Mach profiles are found for discharges with dominant ICRH heating where one expect a similar difference in torque and power deposition profile. It has also been shown that ICRH could drive off-axis momentum in JET, yielding hollow rotation profiles [69]. Typical thermal Mach number profiles are shown in figure 4.4.

4.3.2 Alfvén Mach number

A second dimensionless parameter accounting for plasma rotation can be studied, the Alfvén Mach number. It is defined as the ratio of the toroidal fluid velocity normalised by the Alfvén speed:

\[
M_A = \frac{v_φ}{v_A} = \frac{v_φ}{B_φ/\sqrt{\mu_0\rho}}
\]  

(4.16)

This Mach number is related to the electromagnetic properties of the plasma, and is therefore used in MHD theoretical and numerical work, as for example RWM studies. Interestingly, the thermal and Alfvén Mach numbers are linked by the
Figure 4.5: (a) The profile averaged Alfvén Mach number as a function of $\beta$. (b) The Alfvén Mach number peaking factor as a function of the line-integrated density.

relation:

$$\left( \frac{M_A}{M_{th}} \right)^2 = \frac{1}{2} \frac{n e T_i}{B_0^2 / 2 \mu_0} \sim \beta$$

(4.17)

In JET, $\beta$ values of order $10^{-2}$ are observed. The Alfvén Mach number is thus expected to be one order of magnitude lower than the thermal Mach number, and observed values are in the range $9 \times 10^{-4} \leq \langle M_A \rangle_p \leq 0.07$. Figure 4.5-(a) shows that $\langle M_A \rangle_p$ scales approximately with $\beta$. A similar dependence was found for Alfvén Mach numbers in non-NBI heated plasmas [74]. Nonetheless the trend shown here could also be due to the coupling between the heating and torque sources in these discharges which are predominantly heated by NBI. At comparable magnetic field, a high torque results in a high Alfvén Mach number, while the corresponding important heating gives rise to high temperature, hence $\beta$. A detailed look shows that $\langle M_A \rangle_p$ is also lower for H-modes with type III ELMs compared to those with type I ELMs.

The peaking of the Alfvén Mach profile was also calculated. Comparing the definitions of both Mach numbers (equations 4.15 and 4.16), it is clear that the Alfvén Mach number profile is expected to be more peaked than that of thermal Mach number: the $M_A$ profile peaking is a combination of the rotation and density profile peaking. In general the profiles are more peaked for low density discharges, as shown in figure 4.5-(b). This may be due to the contribution of the density peaking. It can also be explained by a torque deposition located more off-axis, as the NBI penetration is reduced in high density plasmas. The higher density then results in flatter rotation and Mach number profiles. The shape of the Alfvén Mach
profile plays an important role in plasma performance. For example, one of the
damping models presented in [29] predicts that a flat rotation with a central Alfvén
Mach number equal to 0.02 in ITER would stabilise the RWM. While using the
same model with the same central Alfvén Mach number, however with a peaked
profile, the mode can be unstable.

4.3.3 Scalings of Mach numbers

A regression analysis on both Mach numbers was carried out in order to further in-
vestigate their dependence on plasma parameters. The regression analysis consists
in a linear fit to the logarithm of the parameter set. The quality of the fit of the
model to the data is given by the Pearson coefficient of determination $R^2$, which is
unity for a perfect fit. The normalised $\chi^2$, denoted $\chi^2_N$, is also an indicator of the
quality of the fit. Provided the experimental error bars are correctly estimated, it
is greater than 1, with equality for a perfect fit. The regression analysis technique
is detailed in appendix C, where the definitions of $R^2$ and $\chi^2_N$ are given.

The models used to fit the experimental data include the main engineering
parameters that define a tokamak discharge. The best fitting model used the
line-integrated density $n_{el}$, plasma current $I_p$, central toroidal magnetic field $B_{\phi,0}$,
toroidal torque $T_\phi$, and total heating power, $P_{in}$. Reducing the set of parameter
in the model lead to a degradation of the fit, while adding other parameters did
not significantly improve the fit, suggesting that the model mentioned above uses
the optimum parameter set. The obtained scalings are:

$$\langle M_{th} \rangle_p \propto n_{el}^{-0.12\pm0.03} I_p^{+0.49\pm0.06} B_{\phi,0}^{-0.43\pm0.08} P_{in}^{-0.51\pm0.03} T_\phi^{+0.73\pm0.02}$$ (4.18)

$$\langle M_A \rangle_p \propto n_{el}^{-0.08\pm0.04} I_p^{+0.80\pm0.08} B_{\phi,0}^{-1.12\pm0.12} P_{in}^{-0.36\pm0.04} T_\phi^{+0.95\pm0.04}$$ (4.19)

The error bars given here correspond to the standard deviation on the coefficients
within which the regression analyses would produce a fit of equal quality. The
Pearson coefficient of determination and normalised $\chi^2$ for scaling 4.18 are $R^2 = 0.88$ and $\chi^2_N = 1.11$. Those of scaling 4.19 are $R^2 = 0.84$ and $\chi^2_N = 1.14$. The
values of the profile average Mach numbers given by scalings 4.18 and 4.19 are
plotted against the experimental values on figures 4.6-(a) and 4.6-(b). As seen in
table 4.3, $T_\phi$ and $P_{in}$ as well as $I_p$ and $B_{\phi,0}$ are coupled, which can also be seen
on figures 4.1-(b) and 4.1-(c). The coupling is however weak enough to produce
reasonable fits to the data, as shown by the $R^2$ and $\chi^2_N$ values.

The scaling with the density is weakly negative for both the thermal and the
Alfvén Mach numbers. A positive scaling with toroidal torque and a negative
scaling with input power is found. This comes close to the basic trend shown in
figure 4.3-(a) for the thermal Mach number. A slightly different dependence on $T_\phi$
and $P_{in}$ is found for the Alfvén Mach numbers. Both Mach numbers scale quite
strongly with the ratio of $I_p$ to $B_{\phi,0}$, which is approximately proportional to $q_{95}^{-1}$.
The Alfvén Mach number in particular, scales almost linearly with $q_{95}^{-1}$. 68


4.4 Momentum and energy confinement times in JET

4.4.1 General trend

The momentum and energy confinement time in JET are in the range 100ms \( < \tau_\phi, \tau_E < 500\text{ms} \). The momentum and energy confinement times were observed to be equal in several machines \([75, 76]\), which has been considered consistent with the equality of local diffusivities predicted by electrostatic turbulence theory \([73]\).

As seen in section 4.3.1, the scaling of the thermal Mach number with the ratio of torque and input power indicated a certain degree of coupling in JET’s momentum and energy confinement times. This coupling is confirmed by examining the confinement times themselves: figure 4.7-(a) shows that the measured \( \tau_E/\tau_\phi \) ratio in JET is approximately unity. Nevertheless, this ratio varies from discharge to discharge, with \( 0.4 < \tau_E/\tau_\phi < 1.8 \) (figure 4.7-(a)). The \( \tau_E/\tau_\phi \) spread does not seem to be a randomly distributed scatter, and in figure 4.7-(b), \( \tau_E/\tau_\phi \) decreases with \( \langle M_A \rangle_p \), with the ICRH scenario deviating from this trend. This trend is effectively an enhancement of the momentum confinement, respective to that of energy, with rotation. Such an improvement is consistent with the the representation of the momentum flux as the combination of a momentum diffusion with an inward momentum pinch (section 2.2.1): the momentum transport described by this formalism decreases with increasing rotation. Therefore, while the fact that \( \tau_E/\tau_\phi \) is approximately unity seems to indicate that the relative magnitudes of momentum...
and energy transport in JET are consistent with theoretical predictions of electrostatic turbulence [73], the observed variations of this ratio appear to be consistent with the presence of the inward momentum pinch predicted by [13].

### 4.4.2 Engineering parameter scalings

As seen in chapter 2, rotation is the result of a balance between torque sources on the one hand and transport and losses on the other hand. It is thus of interest to carry out a statistical study of momentum confinement, and its energy counterpart, using the database. Regression analyses of the confinement times gave the scalings:

\[
\tau_\phi \propto n_{el}^{+0.47\pm0.05} I_p^{+1.14\pm0.14} B_{\phi,0}^{+0.48\pm0.14} P_{in}^{-0.54\pm0.05}
\]

\[
\tau_E \propto n_{el}^{+0.41\pm0.02} I_p^{+0.76\pm0.08} B_{\phi,0}^{+0.26\pm0.07} P_{in}^{-0.40\pm0.02}
\]

The Pearson coefficient of determination and normalised \(\chi^2\) for these fits are respectively \(R^2 = 0.63, \chi^2_N = 8.54\) and \(R^2 = 0.78, \chi^2_N = 1.53\). The parameter dependences in scaling 4.21 have the same directions as the ones in the scaling 4.13, derived from the multi-machine H-mode confinement database. They are however not identical, which of course is not surprising, since the latter includes more parameters, and given its much larger number of entries as well as its inclusion of various machines with diverse sizes and shapes. Both equations 4.20 and 4.21 show an inverse dependence on input power, which is consistent with turbulent transport theory, since the heating provides the source of free energy for the micro-scale fluctuations involved in the turbulence. Although the \(\tau_E\) fit quality is
CHAPTER 4. DATABASE STUDY OF PLASMA ROTATION AT JET

Figure 4.8: (a) Values of the momentum confinement time predicted by scaling 4.24 plotted against the experimental values. (b) Same plot for the energy confinement time predicted by scaling 4.25. The respective numerical coefficients for these scalings are 1.08 and 0.43 (using the units given in table 4.2).

reasonable (scaling 4.21, with $R^2 = 0.78$, $\chi^2_N = 1.53$), the $\tau_\phi$ one is not (scaling 4.20, with $R^2 = 0.63$, $\chi^2_N = 8.54$), hinting that another parameter should be added to the model. Given its role as a rotation source and the possible role of rotation in confinement, torque was added to the scaling parameters, yielding:

$$
\tau_\phi \propto n_{el}^{+0.48 \pm 0.04} I_p^{+1.03 \pm 0.10} P_{\phi,0}^{-0.15 \pm 0.11} P_{in}^{-0.33 \pm 0.06} T_\phi^{-0.10 \pm 0.03} \quad (4.22)
$$

$$
\tau_E \propto n_{el}^{+0.39 \pm 0.03} I_p^{+0.76 \pm 0.07} P_{\phi,0}^{+0.20 \pm 0.07} P_{in}^{-0.41 \pm 0.04} T_\phi^{+0.08 \pm 0.02} \quad (4.23)
$$

For these scalings, $R^2 = 0.74$, $\chi^2_N = 3.96$ and $R^2 = 0.80$, $\chi^2_N = 1.48$ respectively. The quality of scalings 4.21 and 4.23 are similar, and according to the latter scaling, $\tau_E$ weakly scales with torque. This is consistent with an improvement of confinement by rotation. Scaling 4.22 however, shows a weakly decreasing momentum confinement with torque. The inclusion of torque improves the quality of this fit substantially, while including other parameters did not, indicating that $T_\phi$ is relevant to momentum confinement, which although intuitive, is not a priori obvious.

4.4.3 Mixed and dimensionless parameter scalings

Scalings 4.22 and 4.23 examine the dependence of confinement on the applied torque. The emphasis can be put on rotation itself, rather than on its source, by including the profile average Alfvén Mach number instead of the torque. Such scalings benefit from the fact that, unlike $T_\phi$ and $P_{in}$, $\langle M_A \rangle_p$ and $P_{in}$ are not
correlated (table 4.3). Although theory indicates that rotationnal shear is more relevant to confinement than rotation itself, the former is a local value which proves difficult to include in a scalar database. In addition, quantities associated with gradients suffer from high uncertainties (up to 50%) which deeply affect the quality of the fit. The scaling of the confinement times based on $\langle M_A \rangle_p$ are:

$$
\tau_\phi \propto n_{el}^{+0.39 \pm 0.03} I_p^{+0.79 \pm 0.03} B_{\phi,0}^{+0.13 \pm 0.11} P_{in}^{-0.75 \pm 0.04} \langle M_A \rangle_p^{+0.31 \pm 0.03}
$$

$$
\tau_E \propto n_{el}^{+0.37 \pm 0.02} I_p^{+0.56 \pm 0.06} B_{\phi,0}^{+0.17 \pm 0.06} P_{in}^{-0.48 \pm 0.02} \langle M_A \rangle_p^{+0.21 \pm 0.02}
$$

Here, $R^2 = 0.78$, $\chi^2_N = 3.70$ and $R^2 = 0.86$, $\chi^2_N = 0.99$ respectively. The fit quality for the energy confinement time is much improved with respect to scaling 4.21 when including rotation in terms of $\langle M_A \rangle_p$, possibly confirming the dependence of energy confinement on rotation. This quality improvement was not observed in scaling 4.23, probably because of the correlation between $T_\phi$ and $P_{in}$. It is reassuring that the direction of the parameter dependence of scaling 4.20 and 4.21 are conserved in scalings 4.24 and 4.25. The confinement time values according to the scalings are plotted against the experimental ones in figures 4.8-(a) and 4.8-(b). It can be seen that scaling 4.24 underestimates the momentum confinement time in counter-NBI shots and overestimates it for the predominant ICRH ones. This can indicate that in both these scenarios, either the transport itself differs, or the sources and/or their profile are not correctly accounted for. For the predominant ICRH plasmas, this may be due to an additional torque source, as discussed in [69]. It should be noted that the lower $\langle M_A \rangle_p$ coefficient in scaling 4.25 than in scaling 4.24 is consistent with the trend observed for $\tau_E/\tau_\phi$ in section 4.4.1.

Scalings 4.23 and 4.22 give the dependence of the momentum and energy confinement times upon engineering parameters, which gives them a certain relevance since these parameters are directly controlled in experiments. Although of better quality, scalings 4.25 and 4.24 effectively mix engineering and dimensionless parameters. The dimensional analysis and scale invariance theories presented in sections 4.1.1 provide a strong theoretical background for scalings using dimensionless parameters exclusively. Such scalings are not easy to derive directly, as the dimensionless parameters are difficult to compute systematically and often have a large uncertainty. It is however possible to convert a dimensional $\tau_E$ scaling into
a dimensionless one using transformations based on the relations [64,65]:

\[ \nu_* \propto \frac{n_e q_{95}}{\epsilon_p^{3/2} T^2} R_0 \] (4.26)

\[ \rho_* \propto \frac{T^{1/2}}{B_{\phi,0} \epsilon_p} R_0^{-1} \] (4.27)

\[ \beta \propto \frac{n_e T}{B_{\phi,0}^2} \] (4.28)

\[ q_{95} \propto \frac{B_{\phi,0}^2 \kappa}{I_p} R_0 \] (4.29)

The temperature \( T \) must then be expressed as a function of engineering parameters, which is done by using the \( \tau_{E} \) scaling itself. This yields a direct expression of the dimensionless parameters as a function of the dimensional ones. Details of this transformation, which is simply a matrix inversion, are given in appendix D. This is not rigorously equivalent to directly deriving scalings based on dimensionless parameters, but the consistency of the converted scaling with the principles of dimensional analysis can be assessed in some cases. This assessment consists in making sure that the dimensionless scaling is dimensionally correct, as detailed in section D.2.

Scalings 4.23 and 4.22 could be converted into a power law of \((\nu_*, \rho_*, \beta, q_{95}, \langle M_{th} \rangle_p)\) or \((\nu_*, \rho_*, \beta, q_{95}, \langle M_A \rangle_p)\). This is only possible because a \( \tau_{\phi} \) scaling (equation 4.22) is available to express the rotation as a function of engineering parameters (in the same way that the \( \tau_{E} \) scaling is used to eliminate \( T \)). Alternatively, the Mach numbers could simply be expressed as a function of engineering parameters using scalings 4.18 and 4.19. Both these conversions however rely on scalings affected by the strong correlation of input power and torque (scalings Scalings 4.23 and 4.22). It therefore seems more appropriate to convert scalings 4.25 and 4.24 which do not feature this problem and are of better quality. This means going from parameter set \((n_{el}, I_p, B_{\phi}, P, \langle M_A \rangle_p)\) to \((\nu_*, \rho_*, \beta, q_{95}, \langle M_A \rangle_p)\).

This conversion is described in appendix D and yields the dimensionless scalings:

\[ \omega_{L,i} \tau_{\phi} \propto \nu_*^{-0.02} \rho_*^{-1.98} \beta^{-0.49} q_{95}^{-1.58} \langle M_A \rangle_p^{+0.61} \] (4.30)

\[ \omega_{L,i} \tau_{E} \propto \nu_*^{-0.14} \rho_*^{-2.26} \beta^{-0.07} q_{95}^{-0.94} \langle M_A \rangle_p^{+0.40} \] (4.31)

These scalings may suffer from a correlation between \( \beta \) and \( \langle M_A \rangle_p \) (section 4.3.2) and as explained in section D.2, it is not possible to assess whether they are dimensionally correct. Equation 4.30 indicates a degradation of the momentum confinement time with \( \beta \), and an enhancement with \( \langle M_A \rangle_p \). The momentum confinement time is found to scale negatively with normalised Larmor radius \( \rho_* \), close to a Bohm-type scaling \( \omega_{L,i} \tau_{\phi} \propto \rho_*^{-2} \), associated with transport dominated by mesoscale (\( \sim \sqrt{\rho_{L,i} a} \)) turbulence.
CHAPTER 4. DATABASE STUDY OF PLASMA ROTATION AT JET

The dimensionless scaling of the energy confinement time, scaling 4.31, presents some similarities to the IPB98(y,2) scaling from the international H-mode database (scaling 4.14). The $\nu_*$ dependence is negative and somewhat higher than the that in the IPB98(y,2) scaling, in line with dedicated experiments which found a stronger negative dependence of $\omega_{L,i}\tau_E$ with $\nu_*$ on JET [77] and other machines [78–80]. The $\rho_*$ coefficient is similar to that in the IPB98(y,2) scaling, lying between a Bohm-like and gyro-Bohm-like scaling [81], the latter ($\omega_{L,i}\tau_E \propto \rho_*^{-3}$) being often considered to characteristic of transport induced by microscale ($\sim \rho_{L,i}$) turbulence. The inverse $\beta$ dependence is much weaker than that of the IPB98(y,2) scaling, but agrees with the results of dedicated scaling experiments on DIII-D and JET [66, 82–84]. This almost zero exponent could be indicative of the dominance of electrostatic turbulent transport rather than transport induced by electromagnetic turbulence [82, 85]. The $q_{95}$ dependence in scalings 4.30 and 4.31 have the same directions as that in the IPB98(y,2) scaling, but is quite weaker.

Both confinement times scale positively, and rather strongly, with the Alfvén Mach number. It is difficult to estimate the error inherent in the conversion from which scalings 4.31 and 4.30 were obtained. In particular, no estimate of their dimensional correctness can be obtained (section D.2). Nevertheless, the positive coefficient for $\langle M_A \rangle_p$ seems to indicate an enhancement of confinement time with rotation.

4.5 Summary

This chapter introduced the dimensional analysis and scale invariance theory which are at the basis of the scaling of confinement time in tokamaks. The large database built at JET in order to carry out such analysis was presented, together with an assessment of its quality. It helped characterise plasma rotation, with a thermal Mach number in the range $0.02 < \langle M_{th} \rangle_p < 0.62$ and an Alfvén Mach number an order of magnitude lower. The Mach numbers were found to differ from one plasma scenario to another, with the type III ELMy H-mode having significantly lower Mach numbers. Of course, ICRH plasmas with low torque input featured reduced Mach numbers with respect to other scenarios. The profile shape of Mach numbers varied from highly peaked in low density high temperature discharges, to flat in plasmas with high density and low temperature, or even hollow in some ICRH heated or counter-NBI plasmas. The scalings of the Mach numbers with plasma engineering parameters were derived and confirmed the observed dependences. In particular, both Mach numbers were observed to decrease with density and edge safety factor. Such characterisation of rotation can prove useful, for example to provide indications on MHD stability, especially in the case of the RWM. Some observations of broad trends showed a coupling between the momentum and energy confinement times in JET, in agreement with theoretical predictions. Such
observations included the scaling of the thermal Mach number with the ratio of torque and heating power, or the somewhat constant $\tau_E/\tau_\phi$ ratio. The variations of this ratio with the Alfvén Mach number could also be indicative of the presence of an inward momentum pinch in momentum transport.

As the database only includes plasmas free of macro-scale MHD modes, their dominant transport drive is expected to be turbulence. Scalings of the momentum and energy confinement times with plasma engineering parameters were derived. The scalings of the energy confinement time were found similar to those derived from the international H-mode database, with a positive dependence on density, plasma current and magnetic field, as well as a negative dependence on auxiliary heating power. Theory indicates that rotationnal shear is more relevant to confinement, it is however difficult to account for this quantity in a scalar database. As a result, rotation itself rather than its shear was used. While its inclusion in the form of the toroidal torque did not improve the fits significantly, its inclusion in the form of the Alfvén Mach number led to a better scaling quality. This could indicate that rotation plays a role in confinement. These scalings of confinement times based on engineering parameters were converted in dimensionless parameter scalings including rotation. Although the validity of the conversion could not be assessed, the parameter exponents were found in agreement with results of experiments carried out in different tokamaks. In particular, the dependence on the normalised Larmor radius was found in between a Bohm and gyro-Bohm like scaling, and a negligible $\beta$ dependence seemed to point to an electrostatic nature of turbulence, as opposed to electromagnetic. Positive exponents for the Alfvén Mach number in both the dimensional and dimensionless scalings of confinement times can be interpreted as an indication of the beneficial role of rotation in plasma confinement.
Chapter 5
Damping of core rotation by MHD in MAST

The previous chapter examined the broad trends of rotation and the influence of plasma parameters on toroidal flows. It also investigated the scaling of confinement times with an eye towards identifying a possible dependence on plasma rotation. While transport in plasmas from the JET database was expected to arise from turbulence, the focus of the present chapter is macro-scale MHD instabilities. Rotation does not only influence the stability of the plasma, but can also be affected by perturbations occurring in it as mentioned in chapter 2. This chapter investigates such an interaction taking place in MAST, during which the appearance of a saturated MHD instability leads to the damping of core plasma rotation. The MHD mode is first introduced, and consistently analysed theoretically, experimentally and numerically. After gaining insight on the mode itself, possible mechanisms for the resulting plasma braking are reviewed, and it is shown that Neoclassical Toroidal Viscosity is the theory best suited to explain the observed damping of core rotation. Calculations of the torque according to this theory require information on the full saturated magnetic structure of the mode, the determination of which, using numerical tools and Soft X-Ray measurements, is detailed. The Neoclassical Toroidal Viscosity mechanism and the calculation of the subsequent torque is then explained, results for MAST are shown and conclusions drawn.

5.1 MAST’s Long-Lived Mode

5.1.1 Observation of the mode

Advanced Tokamak (AT) scenarios aim at steady-state plasma operation, from which a cost-effective tokamak fusion power plant would greatly benefit. They have specific safety factor profiles ranging from strongly reversed shear to broad low shear $q$ profiles (appendix B). MAST plasmas with low density and NBI
heating feature a high temperature core with a resistivity low enough to delay the penetration of the inductively-driven current, thus producing an AT-like $q$ profile (figure 5.1).

The $q$ profile is obtained from EFIT equilibrium reconstructions [7], constrained to pressure profiles deduced from Thomson Scattering (TS), CXRS and $Z_{\text{eff}}$ measurements, and the magnetic pitch angle profile from the Motional Stark Effect diagnostic (MSE). In these plasmas, $\Delta q$ decreases slowly to become close to zero for most of the discharge. $\Delta q$ is defined here as the difference between the minimal $q$ value, $q_{\text{min}}$ and 1. Despite the uncertainty inherent in the EFIT reconstruction, there is great confidence that $\Delta q$ stays positive, meaning that the $q$ profile is above 1, since otherwise the sawtooth instability is predicted to be unstable and would then be observed. As $\Delta q$ evolves towards zero, a saturated Long-Lived Mode (LLM) is observed on outboard mid-plane magnetic probe measurements, and persists until the termination of the plasma, meaning for several confinement times or $\sim 10^6$ Alfvén times (figure 5.2). Simultaneously, a degradation of energy confinement occurs, together with and enhanced fast-ion losses, as indicated by counter-viewing bolometer data. During this phase, the core rotation is strongly damped, with the central velocity decreasing from $\sim 200 \text{km.s}^{-1}$ ($M_A \sim 0.2$) to $50 \text{km.s}^{-1}$ ($M_A \sim 0.05$) in $\sim 15$ ms and the angular frequency profile evolves from peaked to fully flat (figure 5.2). In plasmas featuring the LLM, the high NBI heating power ($P_{\text{NBI}} = 1.8 \text{MW}$) helps sustain a high $\beta$ value, with a maximum of $\beta \sim 0.5$ for the discharge. It is worth noting that the LLM is not observed for plasma with lower $\beta$.

Although saturated modes are most often resistive (that is associated with tearing of magnetic surfaces and reconnection to form magnetic islands [86]), experimental data give no evidence of magnetic reconnection. No local flattening is detected on high resolution TS profiles. $\pi$ phase inversion between neighbouring channels of the poloidal Soft X-Ray (SXR) are also characteristic of magnetic...
reconnection (section 5.3.2) and such phase jumps are indeed observed in the presence of NTMs in MAST, as seen in figure 5.3. This is however not the case in discharges with the LLM (figure 5.14). It can therefore be asserted with some confidence, that the LLM is an ideal mode.

5.1.2 Applicable theories

AT-like q profiles are predicted to be ideally MHD-unstable. Theory analysing the reversed shear q profile indicates that it is prone to the \((m, n) = (1, 1)\) internal kink mode [87]. The mode is unstable, even at zero \(\beta\), for \(\Delta q\) under a critical value \(\Delta q_{\text{crit}}\) calculated analytically for low inverse aspect ratio plasmas in [87]. This mode saturates non-linearly if the q profile remains above 1 [88]. The mode being ideal, the saturation occurs when the stabilising field line bending term balances the fluid drive of the mode.

Theory focusing on flatter core q profiles and not restricted to a toroidal mode number of 1 predicts such plasmas to be unstable to low \((m, n)\) internal kink-balloonning instabilities, called infernal modes [89, 90]. The most unstable modes have \(m = n\) in plasmas with \(q_{\text{min}} \sim 1\) like the ones featuring the LLM. These modes also feature a critical value of \(\Delta q\) under which they are destabilised [90]. This critical value, \(\Delta q_{\text{crit}}\) decreases with the \(n\) number, whereas the mode growth rate increases with it at \(\Delta q \sim 0\). Infernal mode require a finite \(\beta\) to be destabilised.
Both the internal kink and infernal modes are very similar in their drive, structure and threshold, differing only in the core magnetic shear and the toroidal mode numbers assumed to derive their respective theories. Since the $q$ profile in LLM plasmas evolves from reversed to flat shear, while $\beta$ increases, it seems unnecessary to distinguish between these modes in the following discussion.

### 5.1.3 Experimental analysis

Experiments were carried out to test the interpretation of the LLM as an internal kink/infernal mode. Reproducing a plasma with the LLM with increasing density showed that the LLM disappears when the density is high enough. Higher density implies a lower temperature, meaning a higher resistivity and a faster diffusion of the non-inductively driven current. In these circumstances, the $q$ profile becomes monotonic and its minimal value below 1, as can be seen from the presence of sawteeth in the plasma, observed on SXR measurements (figure 5.4-(a)).

The value of the toroidal field was experimentally scanned in order to modify the evolution of the safety factor, although this also changed other parameters, for example $\beta$. In low toroidal field discharges, the entire $q$ profile was lower and $\Delta q$ approached zero earlier than in high field discharges, while the overall shape of the $q$ profile did not change significantly. The onset of the LLM was consistently observed earlier despite lower $\beta$ (figure 5.4-(b)), which corroborates the existence of a $\Delta q$ threshold for the triggering of the mode.
Figure 5.4: (a) Line integrated density as a function of time for several plasma discharges, and corresponding observation of the LLM on SXR. The two shots with lowest density (black and red) feature the LLM as seen from the high frequency SXR signal fluctuations from $t = 250\text{ms}$ onwards, while the one with highest density (green) features periodic sawteeth crashes at $t = 282\text{ms}, 305\text{ms}, 323\text{ms}, 241\text{ms}, 350\text{ms}$ and $358\text{ms}$ instead. (b) Onset time for the LLM as a function of the central toroidal magnetic field for discharges with nominally equal plasma current, density and NBI heating. As the toroidal field is increased, the entire $q$ is higher, delaying the onset of the LLM.

The behaviour of the LLM is modified by changes in the NBI heating waveform around its onset time. Figure 5.5 shows such a change for three different plasmas with the same plasma current, toroidal magnetic field and density. The examined time window is located around $250\text{ms}$, the LLM onset time in a similar plasma with continuous heating. In shot 21792, the LLM has set on before the NBI heating is interrupted. During this interruption, the amplitude of the mode decreases due to a reduction in $\beta$, and the mode disappears. As the core gets colder, the resistivity increases enabling the safety factor to evolve below 1. Sawtooth crashes at $t = 263\text{ms}$ and $t = 275\text{ms}$ confirm that $\Delta q$ becomes negative. Once NBI heating starts again, the plasma is stable to the LLM because $q_{\text{min}}$ is below 1, and the mode is consequently not observed. In shot 22080, the LLM is not yet destabilised when the NBI heating is paused, either because $\beta$ is too low or $\Delta q$ is too high. The ohmic window being short, as the NBI heating restarts, $\Delta q$ is still above its critical value, which it crosses at $t = 265\text{ms}$, a time at which the LLM sets on. In shot 22082, the LLM is not yet triggered when the heating stops. $\Delta q$ however continues dropping and reaches $\Delta q_{\text{crit}}$ during the ohmic phase, the LLM appears at $t = 240\text{ms}$, a time at which $\beta$ is still sufficiently high, even in the absence of NBI heating. Once the NBI heating is switched on again, the mode grows in amplitude.
as $\beta$ rises again. Although these are only interpretations of the LLM behaviour with regards to NBI heating waveform, they tend to corroborate the triggering of the mode at a critical value of $\Delta q$ and $\beta$.

The fluctuations of SXR measurements given by an $(m,n) = (1,1)$ internal kink mode were simulated and are consistent with those observed during the early evolution of the LLM. This simulation is explained in more detail in section 5.3.2. As the mode saturates, the LLM structure gradually includes $n > 1$ toroidal components. Assuming that the $n = 1$ and $n = 2$ components of the experimentally observed LLM both resonate at the $q_{\text{min}}$ surface, the relative amplitude of these components can be inferred from the peaks in the SXR spectrogram. These show that the $n = 2$ component is not detected at LLM onset, whereas it appears and grows in amplitude as $\Delta q$ decreases (figure 5.2-(d)). Since theory suggests that the $n = 1$ mode is unstable at larger $\Delta q$ than instabilities with $n > 1$, but that the latter dominate as $\Delta q$ approaches zero, this gives increased confidence in the interpretation of the LLM as an ideal internal kink/infernal mode.

### 5.1.4 Numerical analysis

The linear stability of MAST plasmas with AT-like $q$ profiles was analysed for different toroidal mode numbers with the MISHKA-1 MHD code [91], which is similar to the CASTOR code described briefly in section 2.1.4. For each $n$, the most unstable mode predicted by MISHKA-1 has a kink-ballooning mode structure with dominant $m = n$, a structure similar to that observed experimentally and those predicted by theory. In order to investigate the stability of equilibria with
CHAPTER 5. DAMPING OF CORE ROTATION BY MHD IN MAST

Figure 5.6: (a) The growth rates of the $n = 1$, 2, and 3 modes as a function of $q_{\text{min}}$ based on equilibrium of shot 21781 at mode onset. The profile is varied by scanning the toroidal field given as input to the MISHKA-1 MHD code. (b) The growth rate of the $n = 1$ mode as a function of toroidal rotation at the $q_{\text{min}}$ surface for different poloidal $\beta$, based on the equilibrium and rotation profile shape of shot 21781 at mode onset. For comparison, the measured rotation at LLM onset is indicated by the tick mark on the horizontal axis.

different $\Delta q$, the toroidal field given as input to MISHKA-1 was scaled accordingly. Figure 5.6-(a) shows the growth rate of the $n = 1, 2, 3$ modes as a function of $\Delta q$. It is evident in this figure that $\Delta q_{\text{crit}}$ decreases with $n$, whereas the mode growth rate increases with it at $\Delta q \sim 0$, consistent with the theoretical features of the ideal internal modes presented in section 5.1.2, and experimental observations. Furthermore, analysis also indicates that at low $\beta$, AT equilibria are stabilised by toroidal plasma rotation (figure 5.6-(b)). Since rotation in MAST can be a significant fraction of the ion sound speed, this helps to explain why low $\beta$ AT-like plasmas do not exhibit the LLM.

5.1.5 Conclusions on the nature of the mode

Analytical theory, experimental analysis and modelling provide a consistent insight into MAST’s long-lived mode. They indicate that the LLM can be described as an ideal MHD mode triggered when $\Delta q$, the difference between the minimal $q$ value and 1, is below a critical value. It is an internal mode which can be associated with the internal kink or infernal modes. It has a dominant toroidal mode number of $n = 1$ at its onset, and $n > 1$ components are destabilised as $\Delta q$ approaches zero. Finally, it is only observed in high $\beta$ plasmas, most probably because MAST’s strong toroidal rotation stabilises the mode at low $\beta$ values.

The analysis presented here is not limited to MAST cases, but could also be relevant to several tokamaks exploiting scenarios with hybrid-like, reversed shear $q$ profiles just above an integer value. Ideal saturated $(m, n) = (2, 1)$ modes are
observed in JET’s hybrid plasmas with $q_{\text{min}} > 2$ [92,93]. This so-called “continuous mode” also results in a flattening of core rotation. Slowly growing ideal modes associated with hybrid like $q$ profile and high normalised pressure have also been observed on DIII-D [94] and JT-60U [95], with a simultaneous collapse of the toroidal rotation.

5.2 Possible braking mechanisms

As described in section 5.1.1, strong damping of core rotation is observed after the onset of the LLM, on a timescale much shorter than that of momentum transport (figure 5.2). During this phase, the mode angular frequency does not evolve significantly. After the plasma angular frequency profile has become flat, both the rotation of the plasma and that of the mode gradually decrease.

This study focuses on the initial flattening of the rotation. Its short timescale compared to that of momentum transport implies that the braking of the plasma does not arise from MHD-enhanced momentum transport. The LLM results in fast ion losses and redistribution, which affects rotation. Estimates of the order of magnitude of the torque associated with this effect show that it is significantly smaller than the torque needed to produce the observed braking (appendix E).

Theoretical, experimental and modelling arguments consistently indicate that the LLM is of ideal MHD nature (section 5.1). This rules out resonant electromagnetic torques associated with reconnection [96,97] as a mechanism for plasma braking. Electromagnetic torques can act on the plasma even in the absence of magnetic islands [98]. They are nevertheless strongly localised around the minimal and integer $q$ locations, inconsistent with the collapse of the whole rotation profile caused by the LLM. In plasmas featuring the LLM, the innermost region of the plasma which would be affected by this torque is located around mid-radius. This means that electromagnetic torques would result in a maximal damping outside the core of the plasma, in contradiction with the observations. Significantly affecting core rotation with these torques would require a momentum diffusion about an order of magnitude higher than that observed before mode onset, and the resulting damping profile would still not be peaked in the core, as seen in the experiment.

In contrast, the torque arising from Neoclassical Toroidal Viscosity (NTV) [99] is distributed and occurs on a thermal ion collision time scale ($\sim 10^{-4}$s), thus appearing well suited to describe the measured braking of the plasma by the LLM.

The following sections therefore investigate whether the torque predicted by NTV theory is consistent with the observed damping of core rotation following the LLM onset. The structure of the LLM is first determined using CASTOR, its saturated amplitude is then estimated by comparing results of forward simulations of the SXR emission to the measurements. This is detailed in section 5.3. Based on this information, the NTV torque is calculated and compared to experimental data,
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**Objective**: investigate whether NTV torque is consistent with experimental observations

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Figure 5.7: Rationale of the theory-experiment comparison carried out in this chapter.

Figure 5.7 summarises the rationale of the analysis described through sections 5.3 and 5.4. Section 5.5 presents results of experiments carried out on MAST and draws conclusions.

5.3 **Structure of the LLM**

The structure of the LLM is investigated using the CASTOR code (section 5.3.1). It is a linear code, therefore giving the eigenstructure of the most unstable mode, but without any saturation amplitude. MHD modes can be observed on the SXR diagnostic (section 5.3.2), which here, yields an estimate of the saturated amplitude. This is done, given the eigenstructure calculated by CASTOR, by simulating the observed SXR fluctuations for a range of assumed amplitudes and selecting the best agreement with the experimental data (section 5.3.3 and 5.3.4). The upper part of figure 5.7 summarises this procedure.

5.3.1 **Eigenstructure analysis from the CASTOR code**

The equilibria analysed here are reconstructed using the EFIT code [7], constrained to magnetic field pitch-angle data from the MSE diagnostic, as well as total pressure data measured by TS, CXRS and $Z_{eff}$ measurements, where $Z_{eff}$ is the effective ion charge. This enables an accurate calculation of the $q$ profile of the discharge (figure 5.1). The pressure constraint is relaxed in the core because the contribution of the fast ions, not measured experimentally, can be significant in this region. Figure 5.8 shows the agreement of the measured data with that reconstructed by EFIT for shot 21508. The CASTOR code requires the equilibrium input to be calculated by the HELENA code [100], a fixed boundary Grad-Shafranov equation solver employing the same straight field line coordinates as in the linear stability analysis. To ensure the closest match between the EFIT and HELENA
5.3.2 Observation of MHD on SXR

The SXR cameras collect light in the frequency range $800\text{eV} < h\nu < 10\text{keV}$ . For electron temperatures $\sim 1\text{keV}$, the emissivity of the plasma in this wavelength
region is dominated by bremmstrahlung [101]:

\[ \epsilon_{SXR} \propto \frac{n_e n_i Z_{eff}^2}{\sqrt{T_e}} \int e^{-\nu/h e T_e} d\nu \]  

(5.1)

where \( \nu \) is the frequency of emission. From equation 5.1, it can be seen that the SXR emissivity is approximately a flux function, provided density and \( Z_{eff} \) asymmetries due to strong toroidal rotation are negligible. Equation 5.1 also shows that the SXR emissivity is predominantly dependent on \( T_e \) and increases with it. In the case of MAST horizontal SXR array, the lines of sight along which the light is collected are located on a poloidal cross section, at a fixed toroidal position (figure 5.9). For any given line of sight, the flux surface to which it is tangential is the hottest it encounters, and the path along which the flux surface emission is integrated is the longest due to the tangency. In first approximation, the data measured by an SXR line of sight can therefore be considered to depend on the emission of the flux surface it is tangent to. A plasma with toroidal flow moves across the plane of the SXR cameras, and if it is axisymmetric, the signal they measure is constant over time. In the case of a non-axisymmetric plasma, as the magnetic structure advected by the plasma motion flows past the cameras (figure 5.9), they observe a different tangent flux surface with a different temperature, resulting in the fluctuation of the SXR signal in time. The fluctuation amplitude depends on the difference of SXR emissivity between the innermost and outermost flux surfaces which happened to be tangent to the line of sight during one toroidal turn of the magnetic structure. To first order, the SXR fluctuation is of amplitude proportional to \( (\vec{\nabla} \epsilon_{SXR}) \cdot \vec{\xi} \), where \( \vec{\xi} \) is the displacement from the mode.

This close link between the displacement of the flux surfaces and the fluctuations observed on SXR data provides useful information on the MHD modes present in the plasma. The SXR measurements on MAST have a time resolution of 10\( \mu \)s, allowing the analysis of fluctuations of period < 50kHz. SXR data exhibits an additional feature when a magnetic island is present in the plasma: a characteristic \( \pi \) phase jump is observed between the two neighbouring lines of sight viewing opposite sides of the island. This is because the temperature perturbation changes sign across the island, and consequently, the two lines of sight measure fluctuations of opposite sign. Such \( \pi \) phase jumps are not observed in plasmas with the LLM. Note however that if an island were present and led to local high \( Z \) impurity accumulation, the resulting parasitic line-emission would screen the phase jump and thus prevent the detection of the island.

### 5.3.3 Simulation of SXR measurements

Equation 5.1 is used to express the SXR emissivity as a function of poloidal flux, \( \epsilon_{SXR} = f(\psi) \). Mid-plane TS measurements yield \( T_e(\psi) \), \( n_e(\psi) \) and \( n_i(\psi) \) (as-
Figure 5.9: (a) Poloidal cross-section of a MAST plasma showing the lines of sight of the horizontal SXR array. The nested contours are flux surfaces with normalised poloidal flux lower than 0.95, the curves converging towards the magnetic axis represent lines of constant poloidal coordinates used in the HElena and CASTOR codes. (b) Top view of MAST showing the horizontal SXR array position and a schematic of an MHD mode moving past the SXR lines of sight.

Assuming quasi-neutrality) and $Z_{\text{eff}}(\psi)$ is given by the analysis of bremsstrahlung emission on an equilibrium timescale.

The eigenstructure of the $n = 1$ ideal mode is given by CASTOR. Assuming an arbitrary amplitude makes it possible to calculate the poloidal flux perturbation for the entire three-dimensional plasma: $\delta \psi = f(R, \phi, Z)$ (with $(R, \phi, Z)$ the cylindrical coordinates defined in section A.1). Added to the equilibrium flux, this provides the complete topology of the plasma flux surfaces, $\psi = f(R, \phi, Z)$. Substituting it into the expression of SXR emissivity as a function of poloidal flux, $\epsilon_{\text{SXR}} = f(\psi)$, yields a three-dimensional map of the SXR emissivity of the plasma: $\epsilon_{\text{SXR}} = f(R, \phi, Z)$.

The reconstruction described here is valid for a non-rotating magnetic structure. In reality, the structure moves toroidally due to the plasma rotation, and its motion with respect to the plasma. In order for the MHD mode not to lose coherence, this motion has to be rigid rotation, such that $\psi = f(R, \phi + \omega_{\text{MHD}}t, Z)$, where $\omega_{\text{MHD}}$ is the angular frequency of the mode deduced from the experimental data by Fourier transform and $t$ the time. The magnetic structure therefore moves across the camera plane located at $\phi = \phi_0$ and the SXR emission on that plane is $\epsilon_{\text{SXR}} = f(R, \phi_0 + \omega_{\text{MHD}}t, Z)$. This allows the integration of the SXR emission along the path of the camera line of sight at any given time, yielding the time-dependent simulation of the SXR measurements for an arbitrary toroidal mode number and mode amplitude.
5.3.4 Determination of the LLM saturated amplitude

The studied equilibria have been found unstable to the modes of toroidal numbers $n = 1$ and $n = 2$. Thus, the LLM observed in the plasma is a priori a linear combination of these two modes, $\tilde{\xi} = \tilde{\xi}_{n=1} + \tilde{\xi}_{n=2}$. The former has a dominant poloidal number of $m = 1$ producing SXR fluctuation of frequency $f_{MHD}$ close to that of the instability’s toroidal motion, while the latter showed a dominant $m = 2$ component leading to a fluctuation of frequency $2f_{MHD}$. Due to their respective $m$ spectrum, both resonate on the surface of minimum $q$, which according to section 5.3.2 implies that the $f_{MHD}$ component of the fluctuation has an amplitude proportional to $\nabla \epsilon_{SXR} \cdot \tilde{\xi}_{n=1}$ and that of the $2f_{MHD}$ component is proportional to $\nabla \epsilon_{SXR} \cdot \tilde{\xi}_{n=2}$. This means that the relative amplitude of the $n = 1$ and $n = 2$ modes is the ratio of the $f_{MHD}$ and $2f_{MHD}$ components in the Fourier spectrum of the SXR signal. From the onset of the LLM until several tens of milliseconds later, the $n = 2$ mode is of negligible amplitude, if present at all (figure 5.2-(d)). Although the Mirnov coils spectrogram in figure 5.2-(a) does show an $n = 2$ component, these coils measure the magnetic fluctuations outside the plasma, in contrast to the SXR cameras, which are able to observe the plasma core. In addition, it is likely that the $n = 2$ component in figure 5.2-(a) arises as a non-linear consequence of the $n = 1$ mode. The absence of $n > 1$ modes is also expected from the stability analysis of these modes (section 5.1): the $n = 1$ mode is unstable at larger $q_{min}$ than the $n = 2$ mode, and since the $q$ profile evolves downwards during the discharge, the $n = 1$ mode is always the only unstable mode at the appearance of the LLM, which is the phase analysed here. Consequently, only the $n = 1$ mode is considered in the rest of this study.

The SXR signals are simulated for different mode amplitudes and compared to the experimental data. Since the MHD perturbation does not affect the average SXR signal but only results in its variation, this comparison is based on the SXR temporal fluctuations only. This method reduces the influence of parasitic impurity line emission and does not require the modelling of the absolute level of the plasma SXR emissivity. The simulation having the lowest residuals with respect to the measurements is then considered a good estimate of the mode amplitude. (Should it be necessary to include the $n = 2$ mode in the analysis, the simulation would be carried out with various $n = 1$ and $n = 2$ amplitudes, as well as toroidal phases between these components). It is assumed here that, as the instability saturates, its structure remains identical to the linear one. This assumption, although quite strong, is likely to hold outside the mode resonant surfaces and inertial layer [88, 102].
5.4 Torque according to NTV theory

The saturated MHD structure is determined as described in section 5.3. The braking it induces is calculated using Neoclassical Toroidal Viscosity theory. This theory is introduced in section 5.4.1 and a formulation applicable to MAST plasmas is presented in section 5.4.2.

5.4.1 NTV theory

NTV theory describes the damping of the plasma flow arising from the breaking of axisymmetry [99,103]. The underlying mechanism is most easily understood in collisional plasmas where the dissipation of toroidal angular momentum is similar to that occurring during magnetic pumping [104]. The presence of the magnetic perturbation results in the distortion of the plasma flux tubes. Flux conservation prescribes parts of the flux tube with small cross-section to have a high magnetic field, hence a high perpendicular pressure by conservation of the first adiabatic invariant (the fluid cell contains the same particles over time, provided many collisions occur during its toroidal precession). If the collision time is short compared to the period of the fluid cell motion, constant total pressure on the flux surface indicates that the parallel pressure is low. Conversely, portions of the flux tube...
with large cross section have a low magnetic field, low perpendicular pressure and high parallel pressure. Therefore, as the fluid cell travels across the distorted flux tube, it experiences an oscillation of parallel and perpendicular pressures as well as of flux tube shape. In such a system, the work done by the parallel and perpendicular pressure vanishes because the pressure and shape oscillations are in phase. The effect of collisions is however not instantaneous, and the oscillation of pressures lags that of shape, causing an overall braking of the fluid cell. This mechanism is sketched in figure 5.10. In the collisionless regime more relevant to tokamaks, the variation of the toroidal field results in a drift of the particles trapped in banana orbits. This gives rise to a radial current which exerts a $\vec{j} \times \vec{B}$ torque on the plasma.

A quantitative expression for the NTV force can be obtained by solving the bounce averaged drift-kinetic equation, then taking the velocity moment of the distribution function to obtain the radial flux, and eventually using the flux-friction relation derived from neoclassical theory \[54, 105\] to obtain the plasma viscosity. This derivation is carried out in reference \[99\].

NTV theory has been applied extensively to externally-driven, static magnetic perturbation cases \[106, 107\]. This theory can also be used if the field axisymmetry is broken by the presence of an MHD instability, in which case the torque arises from the differential flow of the plasma through the non-axisymmetric perturbation. In this case, the flow damping brings the rotation of the plasma into agreement with that of the magnetic structure (figure 5.11). The equivalence with the static case is found by moving from the lab frame to that traveling with the MHD instability, a frame change which is possible only because the mode rotates as a rigid body. The angular frequency of this motion results from the interaction of the non-axisymmetric magnetic structure with both the plasma and the conducting external components of the tokamak. This frequency is measured experimentally, so that it is not necessary to calculate the external drag on the magnetic structure. This drag explains why the presence of the internal MHD mode not only leads to angular momentum density redistribution but also to an overall loss of momentum.

Since the magnetic perturbation is not applied externally, its structure has to be calculated using an MHD code. Although there are some uncertainties inherent in this calculation, in contrast to the external field case, it is a self-consistent determination of the magnetic perturbation in contrast to that carried out in the externally applied field case. This self-consistency was recently shown to have a significant impact on the predicted NTV torque \[108\].

5.4.2 Formulation

MAST plasmas are mainly in the so-called $1/\nu$ collisionality regime (figure 5.12), where the particles trapped in banana orbits are collisionless and dominate the radial flux of ions. This regime is characterised by $q\omega_{E \times B} < \nu_{ii} / \epsilon < \sqrt{\epsilon \omega_{l,i}}$ with
Figure 5.11: The two competing angular frequency profiles in a rotating plasma with an MHD mode: that of the plasma (plain line) and that of the mode (dotted line). These profiles are brought in agreement by the damping of the plasma rotation, with a departure of the plasma angular frequency from that of the MHD of order the diamagnetic frequency, \( \omega^*_{NC} \) (dashed line). This procedure is described by equation 5.8.

\( \omega_{E \times B} \) the \( E \times B \) drift frequency, \( \nu_{ii} \) the thermal ion collision frequency, and \( \epsilon \) the local aspect ratio. \( \omega_{t,i} = (R_0 q)^{-1} v_{th,i} \) is the ion transit frequency, \( v_{th,i} \) the thermal ion velocity and \( R_0 \) the major radius of the plasma.

The NTV theory is expressed in straight field line, constant Jacobian, Hamada coordinates \((v, \zeta, \theta_h)\) \[109\], with \( v \) the volume enclosed by the flux surface, \( \zeta \) and \( \theta_h \) the toroidal and poloidal coordinates. From the geometrical flux coordinates \((\psi, \phi, \theta)\) (\( \psi \) being the poloidal flux, \( \phi \) and \( \theta \) the geometrical toroidal and poloidal angles as described in section A.2), they can be obtained by the transformation \[110\]:

\[
\begin{align*}
v (\psi) &= 2\pi \int_0^\psi d\psi' \oint \frac{d\theta'}{B.\nabla \theta'} \\
\zeta (\psi, \phi, \theta) &= \frac{\phi}{2\pi} + F (\psi) \int_0^\psi \left( \frac{1}{R^2} - \frac{1}{R_0^2} \right) \frac{d\theta'}{B.\nabla \theta'} \\
\theta_h (\psi, \theta) &= \left( \oint \frac{d\theta'}{B.\nabla \theta'} \right)^{-1} \left( \int_0^\theta \frac{d\theta'}{B.\nabla \theta'} \right)
\end{align*}
\]

Note that with the conventions chosen here, \( \zeta \) and \( \theta_h \) are periodic, of period 1 (not \( 2\pi \)). In equation 5.3, \( F (\psi) = RB_\phi \) with \( B_\phi \) the toroidal field.
Figure 5.12: Validity of the $1/\nu$ regime for MAST shot 21508 at $t = 255$ms. This regime is characterised by $q\omega_{E\times B} < \nu_i/\epsilon < \sqrt{\epsilon\omega_{ti}}$, where $q\omega_{E\times B}$ is represented by the dotted line, $\nu_i/\epsilon$ by the solid line, and $\sqrt{\epsilon\omega_{ti}}$ by the dashed line.

brackets denote a flux surface average carried out in the following manner:

$$\langle X \rangle = \left( \int \frac{d\theta'}{B_0 \nabla \theta'} \right)^{-1} \left( \int X \frac{d\theta'}{B_0 \nabla \theta'} \right)$$  \hspace{1cm} (5.5)

Reference [110] details some properties of the coordinates built using equations 5.2-5.4 but does not demonstrate that they are the Hamada coordinates. This is done in appendix F. The braking depends on $|\vec{B}|$, the modulus of the total magnetic field, which needs to be written in its Lagrangian form:

$$|\vec{B} (\vec{r} + \vec{\xi})| = |\vec{B}_0 + \delta \vec{B} + (\vec{\xi} \nabla) \vec{B}|$$  \hspace{1cm} (5.6)

$$= |\vec{B}_0| \left( 1 + \sum_{(m,n) \neq (0,0)} \frac{b_{n,m}}{|\vec{B}_0|} e^{2\pi i (m\theta_h - n\zeta)} \right)$$  \hspace{1cm} (5.7)

Here, $\vec{r}$, $\vec{\xi}$ and $\vec{B}_0$ are the position vector, the displacement vector, and the equilibrium magnetic field respectively. By construction, $b_{n,m}$ are the coefficients of the Fourier decomposition of the magnetic perturbation. In equation 5.7, the term $(\vec{\xi} \nabla) \vec{B}$ is most easily calculated by exploiting the equilibrium field axisymmetry and rewriting it as $(\vec{\xi} \nabla) \vec{B} = \vec{B}_0 (\vec{r} + \vec{\xi}) - \vec{B}_0 (\vec{r}) + O (|\xi|^2)$. This calculation is detailed in appendix G and avoids tedious use of the tractable, though often diverging, $\nabla \times (\vec{\xi} \times \vec{B})$ and $\nabla (\vec{\xi} \cdot \vec{B})$ operators.

Rather than the equation given in [99], it is convenient to use the expression for the $1/\nu$ regime NTV torque given in [103]. The latter is obtained by using the force balance equation for the ion fluid to substitute for the gradient of the electrostatic potential appearing in the former equation. While this gradient is
not measured experimentally, the resulting expression of the torque only depends on profiles available from various plasma diagnostics:

\[ t_{\phi,NTV} = K \sum_{(m,n) \neq (0,0)} |nb_{m,n}|^2 W_{m,n} \left( (\omega_\phi - \omega_{^{\ast}NC}) - \omega_{MHD} \right) \] (5.8)

with:

\[ K = 1.74n_e \frac{eT_i}{2\nu_i} R B_\phi e^{3/2} \left( \langle R \rangle B_\phi^{-1} \langle R^{-2} \rangle \right) \] (5.9)

Here, \( T_i, B_\phi, R \) and \( \omega_\phi \) are the ion fluid temperature, the toroidal magnetic field, the major radius and the toroidal angular frequency respectively. The ion temperature and velocity are assumed to be equal to those of the carbon fluid measured by CXRS. \( \omega_{MHD} \) is the angular frequency of the MHD, calculated by Fourier transform of the SXR signal, it accounts for the necessary change from the lab frame to that of the MHD mode. The \( W_{m,n} \) coefficients are given by:

\[ W_{m,n} = \int_0^1 \frac{(F_{mnc}(\kappa))^2 + (F_{mns}(\kappa))^2}{E(\kappa) - (1 - \kappa^2) K(\kappa)} d\kappa^2 \] (5.10)

where \( \kappa \) is a pitch angle parameter defined in [99], \( E(\kappa) \) and \( K(\kappa) \) the complete elliptic integrals of first and second kind. \( F_{mnc}(\kappa) \) and \( F_{mns}(\kappa) \) are defined by:

\[ F_{mnc}(\kappa) = 2 \int_0^{2\arcsin(\kappa)} \sqrt{\kappa^2 - \sin^2(\theta_b/2)} \cos((m - nq) \theta_b) \, d\theta \] (5.11)

\[ F_{mns}(\kappa) = 2 \int_0^{2\arcsin(\kappa)} \sqrt{\kappa^2 - \sin^2(\theta_b/2)} \sin((m - nq) \theta_b) \, d\theta \] (5.12)

\( \omega_{^{\ast}NC} \) is a neoclassical offset angular frequency defined as:

\[ \omega_{^{\ast}NC} = \frac{3.5}{ZeR_{mid}B_p} \left( \frac{dT_i}{dr} \right) \] (5.13)

where \( B_p \) and \( r \) are the poloidal field and minor radius. \( R_{mid} \) is the major radius of outboard mid-plane point of the flux surface. This offset rotation results from the substitution for the gradient of the electrostatic potential mentioned earlier, and its inclusion has been shown to be required in NTV calculations [111].

The NTV torque given by equation 5.8 is proportional to the squared amplitude of the magnetic perturbation, and to the difference of the ion fluid and mode frequencies, \( \omega_\phi - \omega_{MHD} \) (with an additional offset \( \omega_{^{\ast}NC} \)), as expected from the heuristic mechanisms introduced in section 5.4.1.
5.5 MAST results

The comparison described in the previous sections and summarised in figure 5.7 was carried out on MAST plasmas featuring the LLM. The eigenstructure calculated by CASTOR, for an equilibrium reconstructed at the LLM appearance, is an \( n = 1 \) internal kink mode, as mentioned in section 5.1.4, and is shown in figure 5.13-(a). The best agreement between SXR simulations and experimental data is obtained for a radial amplitude \( \xi_r = 1.2 \text{ cm} \) at the mode’s resonant surfaces. The relative amplitudes as well as the phases of the SXR fluctuations are well matched for each channel (figures 5.13-(b), and 5.14-(a)). This best match corresponds to a clear local minimum of the simulation residuals with respect to the measured data (figure 5.14-(b)), which gives good confidence in the estimated amplitude. As mentioned in section 5.3, no \( \pi \) phase jumps are observed in the SXR fluctuations, tending to confirm the ideal MHD nature of the LLM and rule out electromagnetic torques associated with magnetic reconnection.

The rotation frequency profiles of the plasma during the braking are shown in figure 5.15-(a). The rotation is unchanged for the radial location \( R = 1.15 \text{ m} \). It is tempting to interpret this point as the position where the mode frequency and that of the plasma are equal, hence where one would expect the torque applied by the mode to vanish. This is actually not the case, and the dashed line in figure 5.15-(a) indicates the frequency of the mode at its onset. Although it decreases on
equilibrium timescales, this latter frequency does not reach that of the plasma at $R = 1.15m$ at any time. This gap between the rotation frequency of the plasma at $R = 1.15m$ and that of the mode can be explained by the presence of the offset frequency $\omega_{NC}$ in the NTV formulation (equation 5.8), and these two shifts are of similar magnitude. Were this offset not present in the NTV formulation, equation 5.8 would predict an acceleration of the plasma due to the mode outside a major radius of $R = 1.06m$, a value significantly lower than the major radius at which the plasma angular frequency stays constant. This can be considered as an additional indication of the importance of the offset rotation $\omega_{NC}$ in NTV theory, after that provided by [111].

When the LLM appears, the plasma is assumed in a steady state from the point of view of angular momentum transport. This means that the angular momentum input from NBI exactly balances momentum transport and losses. The braking of the plasma takes place on time scales faster than the momentum confinement time ($\sim 50ms$ on MAST), such that the NBI source can still be assumed to balance the momentum transport and losses during the first milliseconds of the braking. Over this period, the changes in rotation are therefore caused solely by the MHD mode. The comparison carried out in this study focuses on this early rotation damping. Analysing later time slices would require an additional assumption on the transport of angular momentum, which cannot confidently be made. The LLM results in fast ion redistribution and losses, the associated torque can, to first approximation, be neglected in this study (appendix E). Nevertheless, this redistribution of fast ions
may modify the NBI torque deposition profile, an effect which is not taken into account here.

The torque predicted by NTV theory for MAST shot 21508 at \( t = 255\text{ms} \) (immediately after LLM onset) is plotted in figure 5.15-(b), together with the measured rate of change of angular momentum for each flux tube. The predictions and observations have the same order of magnitude. The profile shapes are similar, except in the vicinity of the rational surfaces and the inertial layer, the latter being located at the \( q_{\text{min}} \) surface, due to the absence of a \( q = 1 \) surface. In these regions, large parallel magnetic field perturbations result in a high torque which is not observed in experimental data. Since the linear structure used here only differs from the non-linear saturated one at these positions \([88, 102]\), this disagreement is not regarded as invalidating the applicability of the theory to the observations. The uncertainties in the different plasma profiles involved in the calculation are bounded at a level that does not compromise the calculated order of magnitude of the results, nor the shape of the profile. Nonetheless, more detailed comparisons or use of the theory for predictive purposes remain challenging. It is however worth mentioning that the inclusion of the offset frequency \( \omega_{\text{NC}} \) is crucial in order to reproduce the measured torque profile with NTV theory. Furthermore, the Lagrangian term \( (\hat{\xi}, \nabla) \hat{B} \) must be taken into account to predict a torque of magnitude comparable to the one observed experimentally, neglecting it decreases the calculated result by up to 70%, especially in the vicinity of the magnetic axis. The results for the analysis of a different shot (21781 for mode onset at \( t = 255\text{ms} \)) are shown in figure 5.16.

In previous work, NTV has mainly been applied to externally applied magnetic perturbations \([106, 107]\). The similarity between predictions and experimental ob-
Figure 5.16: (a) Profiles of plasma rotation frequency following mode onset for shot 21781. The dashed horizontal line indicates the mode frequency at $t = 255s$. (b) The torque predicted by NTV theory (plain line) and the measured rate of change of angular momentum density (dashed line) as a function of major radius for shot 21781 at $t = 255ms$.

Observations are encouragements that this theory is also a good candidate mechanism for the interaction between MHD and plasma rotation, as described in section 5.4. For this type of study, the theory is applied in the frame moving with the MHD mode, which has a rigid body rotation. This application to an MHD instability also provides an additional evidence for the need of accounting for the offset rotation term $\omega^{*}_{NC}$, after that given by experiments with coil-induced magnetic perturbations on DIII-D [111].

5.6 Summary

This chapter investigated the interaction between a saturated MHD mode, the Long-Lived Mode (LLM), and rotation. The LLM was consistently analysed theoretically, experimentally and numerically. It was shown to be an ideal internal kink mode/infernal mode destabilised in plasmas with reversed shear to broad low shear $q$ profiles. The mode sets on when the difference between the minimal $q$ value and 1 ($\Delta q$) is below a certain critical value. It is stabilised by rotation at low $\beta$, has a predominant toroidal mode number of $n = 1$ at onset, with higher $n$ components appearing as $\Delta q$ decreases further.

The linear structure of the LLM was calculated by the CASTOR MHD code. Its saturated amplitude was determined using simulations of the Soft X-Ray measurements, with the assumption that as the mode saturates, its structure remains similar to the linear one. Based on the determined saturated magnetic structure of the LLM and its measured rotation frequency, the torque according to the Neo-classical Toroidal Viscosity was calculated, and shown to be of the same order of magnitude as the measured rate of change of angular momentum density, and of
similar profile shape. This gave good confidence that Neoclassical Toroidal Viscosity is a credible candidate mechanism to explain the damping of plasma rotation arising from the differential plasma flow through the distorted magnetic structure produced by internal MHD modes.
Chapter 6

Conclusions and discussion

6.1 Conclusions

The work presented here aimed at investigating the toroidal rotation of tokamak plasmas with a particular focus on its interaction with plasma confinement. This was undertaken by setting up a large rotation database of JET MHD-stable plasmas, in order to identify broad trends of rotation. It was shown that the thermal Mach number in this device was in the range $0.02 < M_{th} < 0.62$ and differed with respect to plasma scenarios. The Alfvén Mach number was observed to be one order of magnitude lower than the thermal Mach number in JET. Scalings of both Mach numbers were derived, exhibiting a decrease of these quantities with plasma density and edge safety factor. The profile shape of the Mach numbers is also relevant to the suppression of MHD instabilities or turbulence. It varied from peaked in low density, high temperature plasma, to flat in heavier, colder plasmas, or even hollow in some predominantly ICRH heated or counter NBI discharges. Such characterisation of rotation can prove useful, for example to provide indications on MHD stability, especially in the case of the RWM.

Scalings of the momentum and energy confinement times based upon plasma engineering parameters were derived. They showed a dependence of the energy confinement time similar to the scaling obtained from the international H-mode database. Confinement times scaled positively with density, plasma current and magnetic field, while the scaling coefficient of input power was negative:

$$\tau_\phi \propto n_{ei}^{+0.47\pm0.05} I_p^{+1.14\pm0.14} B_{\phi,0}^{+0.48\pm0.14} P_m^{-0.54\pm0.05}$$

$$\tau_E \propto n_{ei}^{+0.41\pm0.02} I_p^{+0.76\pm0.08} B_{\phi,0}^{+0.26\pm0.07} P_m^{-0.40\pm0.02}$$

Scalings including plasma rotation in the form of torque suffered from the coupling between this parameter and the auxiliary power, due to the predominance of NBI as a heating system. This difficulty was overcome by including plasma rotation as the Alfvén Mach number and led to a significant improvement of the fits, while
keeping the direction of parameter dependences:

\[ \tau_\phi \propto n_{el}^{+0.39\pm0.03} I_p^{+0.79\pm0.03} B_{\phi,0}^{+0.13\pm0.11} P_{in}^{-0.75\pm0.04} \langle M_A \rangle_p^{+0.31\pm0.03} \]

\[ \tau_E \propto n_{el}^{+0.37\pm0.02} I_p^{+0.56\pm0.06} B_{\phi,0}^{+0.17\pm0.06} P_{in}^{-0.48\pm0.02} \langle M_A \rangle_p^{+0.21\pm0.02} \]

According to theory, rotational shear is more relevant to confinement, this quantity however could not be accounted for by a scalar in a satisfactory manner. The dimensional scalings were converted into scaling upon dimensionless parameters:

\[ \omega_{\text{L},i}^{-1} \propto \nu_s^{-0.02} \rho_s^{-1.98} \beta^{-0.49} q_{95}^{-1.58} \langle M_A \rangle_p^{+0.61} \]

\[ \omega_{\text{L},i}^{-1}\tau_E \propto \nu_s^{-0.14} \rho_s^{-2.26} \beta^{-0.07} q_{95}^{-0.94} \langle M_A \rangle_p^{+0.40} \]

The resulting dependencies are consistent with the dimensionless scaling derived from the international H-mode database and/or dedicated experiments on several tokamaks. In particular, the dependence upon the normalised Larmor radius lies in between a Bohm-like and Gyro-Bohm like scaling, while no significant dependence on \( \beta \) was found, which could be indicative of transport dominated by electrostatic turbulence. In both the dimensionless and dimensional scalings of confinement time, the exponent of the Mach number was positive, meaning that rotation could be considered as having a beneficial role in confinement.

While the momentum transport analysed with the JET database is expected to arise from turbulence, tokamak rotation was also investigated from an MHD point of view. To do so, a long-lived MHD instability occuring in MAST was studied. Although the analysis included the rotational stabilisation of the mode, it laid much emphasis on the consequences of the mode on rotation. This MHD mode was shown to be an ideal internal kink/infernal mode occuring in plasmas with reversed shear \( q \) profiles with minimal value just above an integer value. It causes a flattening of the rotation profile, while also driven more unstable with lower toroidal flows at low \( \beta \) values. Neoclassical Toroidal Viscosity theory was applied in the frame co-moving with the magnetic perturbation in order to calculate the braking attributed to the confining magnetic field’s loss of axisymmetry. The torque predicted by Neoclassical Toroidal Viscosity was found to be of similar magnitude to the observed rate of change of angular momentum resulting from the MHD instability. The profile shapes were also in agreement at locations where the structure of the mode was accurately known, meaning outside the integer and minimal \( q \) surfaces. This gives good confidence that Neoclassical Toroidal Viscosity theory can be used to account for the changes in plasma rotation induced by the presence of an MHD mode.
6.2 Further work

Additional work could complement the conclusions presented in the previous section. The database analysis results would greatly benefit from additional experiments to decorrelate the auxiliary heating power from the input torque. This would lead to more reliable scaling laws based on engineering parameters. The inclusion of plasma scenarios other than H-modes, which helped reduce the correlation in these parameters, could possibly become unnecessary, meaning that more consistent analysis based on H-mode plasmas exclusively could be carried out. Deriving dimensional confinement times scalings of higher quality also implies that the dimensionless scalings obtained by conversion would be more reliable.

The statistical study of rotation should be extended to include tokamaks other than JET. A multi-machine database, with various plasma shapes and sizes would yield much more information on tokamak rotation and its parameter dependences. In addition, it would lead to an increased reliability of the derived scaling laws based on engineering parameters. The variation of shape and size parameters would also enable a proper assessment of the dimensional correctness of the dimensionless scalings, a good indicator of their validity.

Neoclassical Toroidal Viscosity theory was found to be a suitable mechanism to describe the interaction between a saturated ideal MHD mode and rotation on MAST. Although it was shown to be a second order effect, the redistribution of fast ions plays a role in the observed changes of the rotation profile. Much insight in the behaviour of toroidal rotation would be gained from an accurate modelling of this effect using appropriate numerical codes, such as the HAGIS drift kinetic code [112]. Lastly, Neoclassical Toroidal Viscosity theory is only able to describe the damping of plasma rotation in the frame co-moving with the magnetic structure of this MHD mode. Complete knowledge of the influence of the mode on the plasma flow can only be gained by measuring the rotation frequency of the mode. This means that the physics of the drag exerted by external components on the magnetic structure resulting from an MHD mode needs to be studied, in order to provide a full understanding of the interaction between plasma rotation and such MHD instabilities.
Appendix A

Coordinate systems

A.1 Orthogonal systems

Several coordinate systems are used to describe the topology of the flux surfaces in a tokamak. The most simple one is the usual cylindrical coordinate system (figure A.1-(a)), which uses the symmetry of the torus in the toroidal direction. The coordinates denoting a location in space are the distance to the torus axis, $R$, the (geometrical) toroidal angle, $\phi$, and the altitude, $Z$. The directions orthogonal to the surfaces of constant coordinates are perpendicular to each other, they define the local orthonormal right-handed basis attached to any point: $(\vec{e}_R, \vec{e}_\phi, \vec{e}_Z)$. 

Figure A.1: (a) The usual cylindrical coordinate system $(R, \phi, Z)$ with its orthonormal right-handed basis $(\vec{e}_R, \vec{e}_\phi, \vec{e}_Z)$. (b) The toroidal coordinate system $(r, \phi, \theta)$ with its orthonormal right-handed basis $(\vec{e}_r, \vec{e}_\phi, \vec{e}_\theta)$. 

There is however no information in the \((R, \phi, Z)\) system about the directions parallel and perpendicular to the flux surfaces, which justifies the introduction of the toroidal coordinate system (figure A.1-(b)). To localise a point, the distance to the magnetic axis, \(r\), the (geometrical) toroidal and poloidal angles, \(\phi\) and \(\theta\), are used. The local orthonormal basis is \((\vec{e}_r, \vec{e}_\phi, \vec{e}_\theta)\). Here \(\vec{e}_r\) is the unit vector contained in the \(\phi = \text{cst}\) plane, orthogonal to the flux surface and pointing outwards. \(\vec{e}_\phi\) is the unit vector orthogonal to the \(\phi = \text{cst}\) plane and \(\vec{e}_\theta\) the unit vector such that the basis is orthonormal and oriented in a right-handed manner. Although intuitive, this local basis is not straight-forwardly analytically defined, because the basis vectors are not orthogonal to the surfaces of constant coordinates.

### A.2 Non-orthogonal systems

![Figure A.2: a) The flux coordinate system \((\psi_N, \phi, \theta)\) with geometrical toroidal and poloidal angles, and its co- and contravariant bases. (b) The straight field line flux coordinate system \((\psi_N, \phi, \theta_s)\) with toroidal geometrical angle, and its co- and contravariant bases.](image)

It can be advantageous not to restrict plasma coordinates to locally orthogonal systems. This enables to define their local basis in a straight-forward manner, for example by using the normalised flux, the geometrical toroidal and the geometrical poloidal angle \((\psi_N, \phi, \theta)\) to locate a point in space. The vectors of the covariant basis are locally orthogonal to lines of constant coordinates and given by \(\left(\vec{\nabla} \psi_N, \vec{\nabla} \phi, \vec{\nabla} \theta\right)\) (figure A.2, left). A vector can be decomposed on this basis...
using its covariant components:

\[ \vec{u} = u_{\psi}^{\text{COV}} \vec{\nabla} \psi + u_{\phi}^{\text{COV}} \vec{\nabla} \phi + u_{\theta}^{\text{COV}} \vec{\nabla} \theta \] (A.1)

The covariant basis is not orthonormal, which makes it difficult to express scalar products and norms without a second basis called the contravariant basis. The direction of the contravariant vectors is defined by the intersection of two surfaces of constant coordinates (figure A.2, left). The contravariant basis \((\vec{e}_{\psi}, \vec{e}_{\phi}, \vec{e}_{\theta})\) is thus given by:

\[ \vec{e}_{\psi} = \frac{\vec{\nabla} \phi \times \vec{\nabla} \theta}{J} \] (A.2)

and circular permutations, with \(J\) defined in equation A.4. A vector is decomposed on this basis using its contravariant components:

\[ \vec{u} = u_{\psi}^{\text{CON}} \vec{e}_{\psi} + u_{\phi}^{\text{CON}} \vec{e}_{\phi} + u_{\theta}^{\text{CON}} \vec{e}_{\theta} \] (A.3)

The use of the subscripts \(\text{COV}\) and \(\text{CON}\) does not comply with usual notations, this is to avoid confusion with the components of the vector in the toroidal coordinate systems \((r, \phi, \theta)\). The normalisation factor \(J\) is the Jacobian, the differential volume element defined by the covariant vectors:

\[ J = \left| \vec{\nabla} \psi \times (\vec{\nabla} \phi \times \vec{\nabla} \theta) \right| \] (A.4)

This normalisation enables a simple expression of the scalar product:

\[ \vec{u} \cdot \vec{v} = u_{\psi}^{\text{COV}} v_{\psi}^{\text{CON}} + u_{\phi}^{\text{COV}} v_{\phi}^{\text{CON}} + u_{\theta}^{\text{COV}} v_{\theta}^{\text{CON}} \] (A.5)

\[ = u_{\psi}^{\text{CON}} v_{\psi}^{\text{COV}} + u_{\phi}^{\text{CON}} v_{\phi}^{\text{COV}} + u_{\theta}^{\text{CON}} v_{\theta}^{\text{COV}} \] (A.6)

This property also allows to express any covariant or contravariant component of a vector as a scalar product:

\[ u_{k}^{\text{COV}} = \vec{u} \cdot \vec{e}^{k} \] (A.7)

\[ u_{k}^{\text{CON}} = \vec{u} \cdot \vec{\nabla} k \] (A.8)

In the \((\psi, \phi, \theta)\) coordinate system, the magnetic pitch angle varies on a flux surface, and the safety factor introduced in section 2.1.3 is defined as the flux surface average of its inverse:

\[ q = \left\langle \frac{\vec{B} \cdot \vec{\nabla} \phi}{\vec{B} \cdot \vec{\nabla} \theta} \right\rangle \] (A.9)
It should be noted that the orthonormal local basis of the toroidal coordinates system described in section A.1 is \((\vec{e}_r, \vec{e}_\phi, \vec{e}_\theta) = \left( \nabla \psi_N |^{-1} \nabla \psi_N, \nabla \phi |^{-1} \nabla \phi, |\vec{e}_\theta|^{-1} \vec{e}_\theta \right)\).

Another class of coordinate systems is field aligned coordinates, or straight field line coordinates. They are coordinates where the magnetic field appears straight on a flux surface, meaning that its pitch angle is constant. The simplest form of such coordinates keeps the geometrical toroidal angle associated with the axisymmetry of the tokamak unchanged, and straighten the field by modifying the poloidal coordinate, which then becomes \(\theta_s\). Starting from the \((\psi_n, \phi, \theta)\) coordinates, \(\theta_s\) is a function of \(\psi_N\) and \(\theta\), such that:

\[
\vec{B} \cdot \nabla \theta_s = \vec{B} \cdot \nabla \psi_N \frac{\partial \theta_s}{\partial \psi_N} + \vec{B} \cdot \nabla \theta \frac{\partial \theta_s}{\partial \theta} \tag{A.10}
\]

\[
= \vec{B} \cdot \nabla \theta \frac{\partial \theta_s}{\partial \theta} \tag{A.11}
\]

The inverse pitch angle becomes:

\[
\frac{\vec{B} \cdot \nabla \phi}{\vec{B} \cdot \nabla \theta_s} = \frac{\vec{B} \cdot \nabla \phi}{\vec{B} \cdot \nabla \theta} \frac{\partial \theta_s}{\partial \theta}^{-1} \tag{A.12}
\]

Straightening the magnetic field requires that the variation of the first term on the flux surface be compensated by the second term. It is of course convenient to maintain the definition of the safety factor as the inverse pitch-angle. This defines the partial derivative of \(\theta_s\) in a unique manner:

\[
\frac{\partial \theta_s}{\partial \theta} = \frac{1}{q} \frac{\vec{B} \cdot \nabla \phi}{\vec{B} \cdot \nabla \theta} \tag{A.13}
\]

Together with the boundary condition \(\theta_s (\psi_N, 0) = 0\), integration of equation A.13 gives \(\theta_s (\psi_N, \theta)\) and completes the construction of the \((\psi_N, \phi, \theta_s)\) straight field line coordinates (figure A.2, right).

The systems described in this section can also be built with \(\sqrt{\psi_N}\) as the flux surface label instead of \(\psi_N\).

### A.3 Hamada coordinates

There are many types of straight field line coordinates. The Hamada coordinates\(^{[109]}\) are one of them, and are used for example in the calculation rotation damping according to Neoclassical Toroidal Viscosity (NTV, chapter 5). Their particular characteristic is that they have a constant Jacobian. The Hamada coordinates and their calculation are detailed in appendix F.
Appendix B

Plasma scenarios for tokamaks

During a tokamak plasma discharge, all controllable plasma parameters are tailored in order to reach optimal conditions for nuclear fusion to occur. Each class of plasma profiles produced with this aim is called a plasma scenario, and several of these scenarios are being developed for the operation of a fusion research device and eventually a fusion power plant.

B.1 H-modes

The baseline scenario for tokamak operation is called high confinement mode, or H-mode. This denomination arose from the experimental discovery that for sufficiently high auxiliary heating power, the plasma enters a state with enhanced confinement properties [113]. The H-mode scenario is well documented and has been the subject of much research. The confinement enhancement is produced by a steepening of the pressure gradient at the edge of the plasma, leading to the appearance of a pressure pedestal. This pedestal is unstable and the pressure gradient periodically relaxes to a lower value, releasing energy from the plasma. These periodical relaxations are called Edge Localised Modes (ELMs) and the energy they eject from the plasma can damage the tokamak wall. Various types of ELMs are observed in tokamak plasmas, the main ones being the type I and type III ELMs. Type I ELMs have a slow frequency (∼20Hz on JET) which increases with input power. They release vast amount of energy into the tokamak vessel. Type III ELMs are significantly smaller, but happen with much larger frequencies (∼200Hz on JET), which is decreasing with input power.

B.2 Advanced scenarios

The poloidal magnetic field in a tokamak is produced by a toroidal current running through the plasma. In baseline tokamak scenarios, the main contribution to the internal toroidal plasma current is inductively driven, meaning that the poloidal
magnetic field can only be maintained for a limited duration, dictated by the maximum current intensity supported by the central solenoid. A cost effective tokamak fusion power plant would however need to operate in steady-state, justifying the development of the advanced scenarios [114–118]. These aim at optimising the self-generated plasma current, the so-called “bootstrap current” [119,120]. Note that these scenarios also feature a pressure pedestal, characteristic of H-mode plasmas.

B.2.1 Steady-state scenario

The steady-state scenario [118] is, as its name suggests, designed to be fully non-inductive. In this scenario, the bootstrap current is driven in regions of high pressure gradients arising from the local suppression of turbulence. These regions are called Internal Transport Barriers (ITB) and their appearance requires a non-monotonic $q$ profile or in other words, a negative core magnetic shear $s = r q^{-1} dq/dr$. Such a $q$ profile can be transiently produced by heating the core of the plasma early in the discharge, thus preventing the plasma current from diffusing into the core, since plasma resistivity scales inversely with temperature. Careful tuning of auxiliary heating and current drive then triggers an ITB which gives rise to a steep pressure gradient and an increased bootstrap current, which in turns sustains the reversed shear $q$ profile. This delicate virtuous circle however relies on a precise, possibly real-time, control of plasma parameters. Eventually, the reversed $q$ profile could be sustained relying on current drive actuators such as electron cyclotron current drive (ECCD) or off-axis NBI, this is nevertheless not routinely done in present tokamaks and not planned for ITER.

B.2.2 Hybrid scenario

The “hybrid” scenario [116] does not require as fine a control of plasma parameters as the steady-state scenario. It aims at exploiting a natural quasi-stationary state of the $q$ profile with respect to current diffusion. It is not a fully non-inductive operation scenario in ITER, but could potentially become so with the use of current drives schemes such as ECCD or off-axis NBI. The characteristics of the hybrid scenario are in between the baseline and steady-state scenario, which explains why it is called hybrid. These plasmas have a broad low shear $q$ profile with central value above 1. This prevents deleterious core MHD instabilities from occurring and leads to operation at high plasma pressure, thus enabling the drive of important bootstrap current fractions.
Appendix C

Ordinary least squares fitting

C.1 Linear model

C.1.1 Fitting method

Ordinary least squares fitting (OLS) is a regression method used to determine the best linear model fitting to a dataset. Examining the dependence of a physical quantity $y$ on a set of $p$ parameters $(x_i)_{i \in \{1,2,..,p\}}$, one can rely on a database including $N$ measurements of $y$, $(y_k)_{k \in \{1,2,..,N\}}$, while varying the parameters, $(x_{ij})_{(i,k) \in \{1,2,..,p\} \times \{1,2,..,N\}}$. OLS exploits the linear nature of the problem, and it is naturally most convenient to use matrix notation, arranging the physical quantities in an $N$ column vector $Y$, and an $p \times N$ matrix $X$. The objective here is to find a linear model of $y$ in the form:

$$y^{fit} = \sum_{i=1}^{p} \alpha_i x_i \quad (C.1)$$

which can represent the dependence of the measured $y$ values on the varied $x_i$ parameters. For the $N$ measurements carried out and assuming a value for each of the $\alpha_i$, the fitted values of $y$ can be represented by an $N$ column vector, given by:

$$Y^{fit} = X \alpha \quad (C.2)$$

where $\alpha$ is a $p$ column vector containing the $\alpha_i$. The agreement between the fit and the measurements can be measured by the squared residual, meaning the squared difference between model and measured values summed over the $N$ entries. This is simply:

$$\mathcal{RES}(\alpha) = (Y - X \alpha)^T (Y - X \alpha) \quad (C.3)$$
where the $T$ superscript denotes matrix transposition. The best model is the set of coefficients $\alpha$ with minimal squared residual. The $\mathcal{R}\mathcal{E}\mathcal{S}$ functional is positive and:

\[
\mathcal{R}\mathcal{E}\mathcal{S} (\alpha + d\alpha) = (Y - X\alpha - Xd\alpha)^T (Y - X\alpha - Xd\alpha) = \mathcal{R}\mathcal{E}\mathcal{S} (\alpha) - Y^T Xd\alpha + (X\alpha)^T Xd\alpha + O (|d\alpha|^2) = \mathcal{R}\mathcal{E}\mathcal{S} (\alpha) - 2 (Y^T X + (X\alpha)^T X) d\alpha + O (|d\alpha|^2)
\]

indicating that $\mathcal{R}\mathcal{E}\mathcal{S}$ is minimal for $-Y^T X + (X\alpha)^T X = 0$, that is to say $\alpha = (X^T X)^{-1} X^T Y$, which completely determines the model.

### C.1.2 Fit quality

The squared residual $\mathcal{R}\mathcal{E}\mathcal{S}$ accounts for the quantitative agreement of the fit with the measured data, but depends on the size of the database, $N$. Normalising it by the variance of $y$ over the database, $\sigma^2_y$, and subtracting it from 1 gives a more relevant measure of the fit’s quality, called the Pearson coefficient of determination:

\[
R^2 = 1 - \frac{\mathcal{R}\mathcal{E}\mathcal{S} (\alpha)}{\sigma^2_Y} \tag{C.7}
\]

A residual-free, perfect, fit has $R^2 = 1$ while a fit with squared residual equal to the variance of $y$ over the database has a zero Pearson coefficient of determination.

Another measure of the model’s quality is the normalised $\chi^2$, denoted $\chi^2_N$. It compares the difference between the fit and the measured value with the measurement error on the physical quantity, $\Delta y_i$:

\[
\chi^2_N = \frac{1}{N - p} \sum_{k=1}^{N} \frac{(y_k - \sum_{i=1}^{p} \alpha_i x_{ik})^2}{\Delta y_k^2} \tag{C.8}
\]

Using a $N - p$ normalising factor rather than $N$ corrects for the fact that a perfect fit to the measurements can always be obtained if the number of parameters equals the number of samples, $N - p = 0$. A poor fit will result in a $\chi^2_N$ value significantly larger than 1, while the best achievable fit is attained for $\chi^2_N = 1$, a value at which the difference between the model and the measurements is comparable to the measurement error. Having $\chi^2_N < 1$ means that the model is over-accurate, or that the measurement error is underestimated.
C.2 Log-linear model

The model used in chapter 4 is not linear, but assumes a power dependence of physical quantities on parameters:

$$y_{\text{fit}} = e^{\alpha_0} \prod_{i=1}^{p} x_i^{\alpha_i}$$  \hspace{1cm} (C.9)

This is called a log-linear model. The equivalence with the linear model is found by simply taking the natural logarithm of this relation:

$$\ln(y_{\text{fit}}) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \ln(x_i)$$  \hspace{1cm} (C.10)

Applying the linear model to the logarithm of the studied quantities and of the parameters enables to determine the $\alpha$ coefficients. Note however that because of the $e^{\alpha_0}$ coefficient in model C.9, a constant parameter equal to $e$ has to be added to the $(x_i)_{i \in \{1,2,..p\}}$ parameters to determine $\alpha_0$. This extra parameter needs to be removed in the principal component analysis described in section C.3.

C.3 Principal component analysis

The covariance matrix $C$ is the $p \times p$ square matrix giving the covariance between parameters, $c_{ij} = \text{COV}(x_i, x_j)$. Normalising the coefficients of $C$ by the standard deviations of each parameters gives the correlation matrix $\Gamma$, $\Gamma_{ij} = c_{ij}/\sqrt{c_{ii}c_{jj}}$. The correlation matrix is convenient to account for parameter correlations in the database, with a $|\Gamma_{ij}|$ value close to 1 reflecting a correlation between $x_i$ and $x_j$.

To confidently carry out regression analyses on a database, the latter needs to offer a sufficient decorrelation in parameters as well as a significant spread for each of them. An assessment of these qualities is provided by principal component analysis, the study of the covariance matrix $C$ using the symmetric and bilinear properties of the covariance function. Symmetry implies that the covariance matrix is real symmetric, hence diagonalisable with orthogonal eigenvectors. Thus, there exists an orthogonal matrix $O$ such that $O^T CO = D$, with $D$ a diagonal matrix. The bilinearity of the covariance function guarantees that $D$ is the covariance matrix of a new set of parameters $(x'_i)_{i \in \{1,2,..p\}}$ obtained from the original set by the transformation:

$$x'_i = \sum_{j=1}^{p} o_{ij} x_j$$  \hspace{1cm} (C.11)

$D$ being diagonal, $\text{COV}(x'_i, x'_j; j \neq i) = 0$ and two distinct parameters of the new
parameter set are uncorrelated. The spread of $x'_i$ over the database is given by $d_{ii}$, or more precisely $\sigma_{x'_i} = \sqrt{d_{ii}}$. A large value of $\sqrt{d_{ii}}$ compared to the measurement error of $x'_i$ therefore means that the database offers a wide variation in $x'_i$, and that dependence on $x'_i$ obtained by regression analysis can confidently be considered as relevant. In this case, $x'_i$ is called a well-conditioned component. In contrast, a low value of $\sqrt{d_{ii}}$ indicates a restricted variation in $x'_i$, hence a certain degree of correlation between the $(o_{ij} x_j)_{j \in \{1,2,...p\}}$. Since in practice, the original parameter set has a clear physical meaning whereas the new one rarely has, principal component analysis is only used to assess the quality of the database. If the number of well-conditioned component is close to the number of parameters, correlations in between parameters is unlikely to be detrimental to the results of the regression analysis. Principal Component Analysis carried out on the entire JET database revealed that it has three well conditioned component, while the subset of H-mode entry only has two.
Appendix D

Scaling conversion

D.1 Conversion method

It is possible to convert a dimensional scaling into a dimensionless scaling by examining the proportionality relation between the dimensional parameter set and the dimensionless one. Here the example of the conversion of a $\tau_E$ and $\tau_\phi$ scaling based on $(n_{el}, I_p, B_\phi, P_{in}, \langle M_A \rangle_p)$ into a $\omega_{L,i}\tau_E$ and $\omega_{L,i}\tau_\phi$ scaling depending on $(\nu_*, \rho_*, \beta, q_{95}, \langle M_A \rangle_p)$ is detailed, other conversions being similar.

The dimensionless parameters have the following dependence:

\begin{align*}
\nu_* &\propto \frac{n_{el}q_{95}}{\epsilon_p^3/2T^2} R_0 \quad \text{(D.1)} \\
\rho_* &\propto \frac{T^{1/2}}{B_0 \epsilon_p} R_0^{-1} \quad \text{(D.2)} \\
\beta &\propto \frac{n_e T}{B_0^2} \quad \text{(D.3)} \\
q_{95} &\propto \frac{B_0 \epsilon_p^2 \kappa}{I_p} R_0 \quad \text{(D.4)}
\end{align*}

There remains to eliminate the temperature $T$ in these relations. This is done using the definition of the energy confinement time:

$$\tau_E = \frac{W_{\text{kin}}}{P_{\text{in}}} \propto \frac{n_{el} TV}{P_{\text{in}}}$$ \quad \text{(D.5)}

The volume $V$ can be expressed as $V = 2\pi R_0 \pi a^2 \kappa = 2\pi^2 \epsilon_p^2 \kappa R_0^3$. Therefore:

$$T \propto n_{el}^{-1} \frac{P_{\text{in}}}{\tau_E \epsilon_p^{-2} \kappa^{-1} R_0^{-3}}$$ \quad \text{(D.6)}

In the case of the JET database, the geometrical parameters $\epsilon_p$, $\kappa$ and $R_0$ are almost constant, such that it is irrelevant to include them in scaling laws. It should
however be noted that knowing the dependence on the only remaining dimensional parameter $R_0$ would provide useful information on its consistency with dimensional analysis (section D.2). Ignoring $\epsilon_p$, $\kappa$ and $R_0$, the dependences of the dimensionless parameters are:

\[
\begin{align*}
\nu_* &\propto n_{el}^3 I_p^{-1} B_\phi^1 P_{in}^{-2} \tau_E^{-2} \\
\rho_* &\propto n_{el}^{-\frac{1}{2}} I_p^0 B_\phi^{-1} P_{in}^{-\frac{1}{2}} \tau_E^2 \\
\beta &\propto n_{el}^0 I_p^0 B_\phi^{-2} P_{in}^1 \tau_E^1 \\
q_{95} &\propto n_{el}^0 I_p^{-1} B_\phi^1 P_{in}^0 \tau_E^0 
\end{align*}
\]

(D.7)

Substituting for the energy confinement time using a scaling of the form:

\[
\tau_E \propto n_{el}^{\alpha_n} I_p^{\alpha_I} B_\phi^{\alpha_B} P_{in}^{\alpha_P} \langle M_A \rangle_p^{\alpha_M}
\]

(D.8)

this gives the dimensionless parameters as a function of the initial parameter set:

\[
\begin{align*}
\nu_* &\propto n_{el}^{3-2\alpha_n} I_p^{1-2\alpha_I} B_\phi^{1-2\alpha_B} P_{in}^{-2-2\alpha_P} \langle M_A \rangle_p^{-2\alpha_M} \\
\rho_* &\propto n_{el}^{-\frac{1}{2}+\frac{1}{2}\alpha_n} I_p^{\frac{1}{2}\alpha_I} B_\phi^{-1+\frac{1}{2}\alpha_B} P_{in}^{\frac{1}{2}+\frac{1}{2}\alpha_P} \langle M_A \rangle_p^{\frac{1}{2}\alpha_M} \\
\beta &\propto n_{el}^{\alpha_n} I_p^{\alpha_I} B_\phi^{-\alpha_B} P_{in}^{1+\alpha_P} \langle M_A \rangle_p^{\alpha_M} \\
q_{95} &\propto n_{el}^0 I_p^{-1} B_\phi^1 P_{in}^0 \langle M_A \rangle_p^1 \\
\langle M_A \rangle_p &\propto n_{el}^0 I_p^0 B_\phi^0 P_{in}^0 \langle M_A \rangle_p^0
\end{align*}
\]

(D.9)

If the dimensionless scaling is written in the form:

\[
\omega_{L,\tau E} \propto \nu_*^{\alpha_*} \rho_*^{\alpha_*} \beta^{\alpha_B} q_{95}^{\alpha_B} \langle M_A \rangle_p^{\alpha_M} (D.10)
\]

meaning:

\[
\tau_E \propto \nu_*^{\alpha_*} \rho_*^{\alpha_*} \beta^{\alpha_B} q_{95}^{\alpha_B} \langle M_A \rangle_p^{\alpha_M} B_\phi^{-1} (D.11)
\]

it is possible to substitute for the dimensionless parameters using relation D.9 and identify the exponents of the dimensional parameters. This gives:

\[
\begin{align*}
\alpha_n &= (3 - 2\alpha_n) \alpha_\nu + \left(-\frac{1}{2} + \frac{1}{2}\alpha_n\right) \alpha_\rho + \alpha_\beta + 0\alpha_q + 0\alpha_{M*} \\
\alpha_I &= (-1 - 2\alpha_I) \alpha_\nu + \frac{1}{2} \alpha_I \alpha_\rho + \alpha_I \alpha_\beta - \alpha_q + 0\alpha_{M*} \\
\alpha_B &= (1 - 2\alpha_B) \alpha_\nu + \left(-1 + \frac{1}{2}\alpha_B\right) \alpha_\rho + \left(-2 + \alpha_B\right) \alpha_\beta + 0\alpha_q + 0\alpha_{M*} - 1 \\
\alpha_P &= (-2 - 2\alpha_P) \alpha_\nu + \left(\frac{1}{2} + \frac{1}{2}\alpha_P\right) \alpha_\rho + \left(1 + \alpha_P\right) \alpha_\beta + 0\alpha_q + 0\alpha_{M*} \\
\alpha_M &= -2\alpha_M \alpha_\nu + \frac{1}{2} \alpha_M \alpha_\rho + \alpha_M \alpha_\beta + 0\alpha_q + 0\alpha_{M*}
\end{align*}
\]

(D.12)

Incidently, this system is easily solved if one is interested in the conversion of \(\left(\nu_*, \rho_*, \beta, q_{95}, \langle M_A \rangle_p\right)\) into \(\left(n_{el}, I_p, B_\phi, P_{in}, \langle M_A \rangle_p\right)\). To obtain the reciprocal conversion, it is convenient to write the exponent of \(\left(n_{el}, I_p, B_\phi, P_{in}, \langle M_A \rangle_p\right)\) in a col-
umn vector \(X_D\) and those of \((\nu_*, \rho_*, \beta, q_{95}, \langle M_A \rangle_p)\) in a column vector \(X_*\). Defining the matrix:

\[
M = \begin{pmatrix}
(3 - 2\alpha_n) & (-\frac{1}{2} + \frac{1}{2}\alpha_n) & \alpha_n & 0 & 0 \\
(-1 - 2\alpha_I) & \frac{1}{2}\alpha_I & \alpha_I & -1 & 0 \\
(1 - 2\alpha_B) & (-1 + \frac{1}{2}\alpha_B) & (-2 + \alpha_B) & 1 & 0 \\
(-2 - 2\alpha_P) & (\frac{1}{2} + \frac{1}{2}\alpha_P) & (1 + \alpha_P) & 0 & 0 \\
-2\alpha_M & \frac{1}{2}\alpha_M & \alpha_M & 0 & 1 \\
\end{pmatrix}
\]  
(D.13)

the system D.12 can be written in the matricial form:

\[
X_D = MX_* - \begin{pmatrix}
0 \\
0 \\
\end{pmatrix}
\]  
(D.14)

such that the vector of exponents for the dimensionless parameters is:

\[
X_* = M^{-1} \begin{pmatrix}
X_D + \begin{pmatrix}
0 \\
0 \\
\end{pmatrix}
\end{pmatrix}
\]  
(D.15)

Note that for the conversion of the \(\tau_\phi\) scaling, the same \(M\) matrix should be used, meaning an \(M\) matrix with the coefficients of the \(\tau_E\) scaling, since the conversion is based on the elimination of the temperature \(T\) using the energy confinement time.

### D.2 Consistency with dimensional analysis theory

Section D.1 describes the conversion of a scaling based on \((n_{el}, I_p, B_\phi, P_{in}, \langle M_A \rangle_p)\) into one depending on \((\nu_*, \rho_*, \beta, q_{95}, \langle M_A \rangle_p)\). However, the dependences of the dimensionless parameters given in equations D.1-D.4 include the geometrical parameters \(\epsilon_p, \kappa\) and \(R_0\) which were ignored because they do not vary significantly over the JET rotation database. Ideally one should convert the scalings from \((n_{el}, I_p, B_\phi, P_{in}, \langle M_A \rangle_p, \epsilon_p, \kappa, R_0)\) to \((\nu_*, \rho_*, \beta, q_{95}, \langle M_A \rangle_p, \epsilon_p, \kappa, R_0)\) using a simi-
lar method. This would yield a $\tau_E$ scaling of the form:

$$\omega_{L,i}^{\tau_E} \propto \nu_*^{\alpha \nu} \rho_*^{\alpha \rho} \beta^{\alpha \beta} q_{95}^{\alpha \nu} \left( M_A \right)_p^{\alpha \nu} \epsilon_p^{\alpha \nu} \kappa^{\alpha \kappa} R_0^{\alpha R} \tag{D.16}$$

Scaling D.16 should not depend on the units in which the problem is formulated, in accordance with the principle stated by the Vaschy-Buckingham theorem introduced in section 4.1.1. This implies that $\alpha_R$ should be zero, which is a good indication on the validity of the converted scaling D.16. When $\alpha_R \sim 0$, the scaling is said to be dimensionally correct. If the $R_0$ parameter had varied significantly over the JET rotation database, it would have been possible to make a scaling conversion from engineering parameters to dimensionless parameters and assess whether the converted scaling is consistent with dimensional analysis principles. Unfortunately, the JET rotation database is a single machine database, hence does not provide such an opportunity.
Appendix E

Estimate of the fast ion redistribution torque

The long-lived mode is observed to incur redistribution and loss of fast ions. The losses can be seen on the bolometer diagnostic, which is able to measure the energy escaping from the plasma by radiation and by loss of neutral particles. Plasma radiation is somewhat isotropic, whereas fast particles are lost preferentially in the beam direction. A difference of intensity measured between the bolometer channels viewing in the co- and counter-beam directions therefore indicates fast ion losses.

Measurements of the neutron production rate can also provide an indication of fast ion redistribution and losses. The relatively low ion temperatures in MAST do not enable significant rate of thermal fusion, such that the observed neutrons are born from fusion reaction occurring between beam and thermal ions. In particular, the neutron production rate depends on the fast ion density, \( n_f \). This density results from a balance between the fast ion source (NBI in the case of MAST), transport and sinks (thermalisation or loss by charge exchange recombination with thermal ions). Such a balance and the resulting neutron production rate can be simulated numerically, for example with the TRANSP code \[12\].

An estimate of the level of fast ion redistribution resulting from the LLM can be obtained by comparing the measured neutron rate and that predicted by TRANSP. In simulations which do not take into account the presence of the LLM except for its measured influence on the plasma temperature and density profiles, the predicted neutron rate is significantly lower than that measured. In contrast, TRANSP simulations with an anomalous fast ion diffusion of coefficient \( D_f \sim 0.5\text{m}^2\text{s}^{-1} \) predict a neutron rate comparable with that experimentally observed. This indicates that a diffusive process with \( D_f \sim 0.5\text{m}^2\text{s}^{-1} \) produces a fast-ion redistribution of the same order of magnitude than that produced by the LLM, a mechanism which is used to estimate the resulting torque. The anomalous diffusion is only applied to ions of energy higher than 40keV.
APPENDIX E. EST. OF FAST ION REDISTRIBUTION TORQUE

The fast ion flux is given by:

\[ \vec{\Gamma}_f = -D_f \vec{\nabla}n_f \]  

(E.1)

This gives rise to a current density:

\[ \vec{j}_f = -eD_f \vec{\nabla}n_f \]  

(E.2)

The fast ion current arising from the redistribution is balanced by a return current in the plasma in order to preserve quasi-neutrality. This return current produces a force, in a mechanism similar to that of the prompt NBI torque deposition described in section 2.2.2. The resulting local force density is:

\[ \vec{f}_{j \times B} = eD_f \vec{\nabla}n_f \times \vec{B} \]  

(E.3)

The torque density averaged over the flux surface is therefore:

\[ t_\phi(\psi_N) = e \left< RD_f \frac{\partial n_f}{\partial \psi_N} \left( \vec{\nabla} \psi_N \times \vec{B} \right) \cdot \vec{e}_\phi \right> \]  

(E.4)

where the flux surface average \( \left< \cdot \right> \) is given by equation 5.5.

This torque can be integrated over the region where the plasma is observed to be braked by the LLM, \( R < 1.15 \text{m} \). This amounts to a braking torque of about \( T_{\phi,f} = -0.02 \text{N.m} \). The measured rate of change of momentum density integrated over the same region is \( \frac{\partial L_{\phi,\text{core}}}{\partial t} = -0.22 \text{N.m} \). Although this calculation is only an estimate of the fast ion redistribution torque with limited accuracy, this gives confidence that the effect of fast ion redistribution can only be a second order effect in the rotation damping observed in presence of the LLM.
Appendix F

Hamada coordinates

The Hamada coordinates \((v, \zeta, \theta_h)\) [109] have the following properties:

**Property 1** The first coordinate is the volume enclosed by the flux surfaces.

**Property 2** They are straight field line coordinates, meaning that:

\[
\left( \hat{B} \cdot \nabla \zeta \right) \left( \hat{B} \cdot \nabla \theta_h \right)^{-1} = q(v)
\]

and this ratio is equal to the usual safety factor, which in usual geometrical flux coordinates is:

\[
q(v) = (2\pi)^{-1} \oint \left( \hat{B} \cdot \nabla \phi \right) \left( \hat{B} \cdot \nabla \theta \right)^{-1} d\theta
\]

**Property 3** All the contravariant components of the magnetic field are flux functions.

**Property 4** Their Jacobian is constant and unity.

These properties uniquely define the Hamada covariant basis, hence the full coordinates, since: \(\nabla v\) is fully determined by property 1; properties 2 and 3 set the relative angles and magnitudes of \(\nabla \zeta\) and \(\nabla \theta_h\); and lastly, property 4 sets the relative angles and magnitudes of \(\nabla v\) on the one hand and \(\left( \nabla \zeta, \nabla \theta_h \right)\) on the other hand. It is shown in this section that the coordinates \((v, \zeta, \theta_h)\), given by equations 5.2-5.4, satisfy these four properties and therefore are the Hamada coordinates.

It should first be mentioned that the integral \(\oint X \left( \hat{B} \cdot \nabla \theta' \right)^{-1} d\theta'\) is left unchanged by any monotonic periodic variable change:

\[
\theta' = f(\eta) \Rightarrow \left\{ \begin{array}{l}
d\theta' = f'(\eta) \, d\eta \\
\n\n\end{array} \Rightarrow \oint X \frac{d\theta'}{\hat{B} \cdot \nabla \theta'} = \oint X \frac{d\eta}{\hat{B} \cdot \nabla \eta} \right. \quad \text{(F.1)}
\]

While not directly relevant to this demonstration, this property interestingly shows that any monotonic periodic poloidal coordinate can be used in place of \(\theta\) to build the \((v, \zeta, \theta_h)\) coordinates.
Property 1 is a simple matter of definition. To prove the validity of properties 2-4, it is useful to note a few points beforehand. To begin with, the definition of the volume in equation 5.2 comes from the basic properties of the magnetic field, allowing one to express it as \( \mathbf{B} = F(\psi) \hat{\nabla} \phi + \hat{\nabla} \psi \times \hat{\nabla} \phi \). Since \( \hat{\nabla} \theta \cdot \hat{\nabla} \phi = 0 \), this gives:

\[
dV = \frac{d\psi \, d\phi \, d\theta}{\hat{\nabla} \psi \cdot (\hat{\nabla} \phi \times \hat{\nabla} \theta)} \quad \text{(F.2)}
\]

\[
dV = \frac{d\psi \, d\phi \, d\theta}{\hat{\nabla} \theta \cdot (\hat{\nabla} \psi \times \hat{\nabla} \phi)} \quad \text{(F.3)}
\]

\[
dV = \frac{d\psi \, d\phi \, d\theta}{\mathbf{B} \cdot \hat{\nabla} \theta} \quad \text{(F.4)}
\]

and therefore:

\[
v(\psi) = 2\pi \int_{0}^{\psi} d\psi' \int \frac{d\theta'}{\mathbf{B} \cdot \hat{\nabla} \theta} \quad \text{(F.5)}
\]

In addition, the third covariant vector of the \((v, \zeta, \theta_h)\) coordinates is:

\[
\hat{\nabla} \theta_h = \frac{\partial \theta_h}{\partial \psi} \hat{\nabla} \psi + \frac{\partial \theta_h}{\partial \theta} \hat{\nabla} \theta \quad \text{(F.6)}
\]

\[
\hat{\nabla} \psi = (\oint \frac{d\theta}{\mathbf{B} \cdot \hat{\nabla} \theta})^{-1} \hat{\nabla} \theta \quad \text{(F.7)}
\]

Hence:

\[
\mathbf{B} \cdot \hat{\nabla} \theta_h = \frac{\partial \theta_h}{\partial \psi} \mathbf{B} \cdot \hat{\nabla} \psi + \left( \oint \frac{d\theta}{\mathbf{B} \cdot \hat{\nabla} \theta} \right)^{-1} \mathbf{B} \cdot \hat{\nabla} \theta \quad \text{(F.8)}
\]

\[
\mathbf{B} \cdot \hat{\nabla} \theta_h = \left( \oint \frac{d\theta}{\mathbf{B} \cdot \hat{\nabla} \theta} \right)^{-1} \quad \text{(F.9)}
\]

Note that this equation also shows that \( \mathbf{B} \cdot \hat{\nabla} \theta_h \) is a derivative of the volume. Additionally, it follows from equation F.6 that:

\[
\mathbf{B} \cdot \hat{\nabla} \theta_h = \frac{\partial \theta_h}{\partial \theta} \mathbf{B} \cdot \hat{\nabla} \theta \quad \text{(F.10)}
\]

This means successively:

\[
\frac{\hat{\nabla} \theta}{\mathbf{B} \cdot \hat{\nabla} \theta} = \frac{\hat{\nabla} \theta_h}{\mathbf{B} \cdot \hat{\nabla} \theta} + \frac{\partial \theta}{\partial \psi} \frac{\hat{\nabla} \psi}{\mathbf{B} \cdot \hat{\nabla} \theta} \quad \text{(F.11)}
\]

\[
\frac{\hat{\nabla} \psi \times \hat{\nabla} \theta}{\mathbf{B} \cdot \hat{\nabla} \theta} = \frac{\hat{\nabla} \psi \times \hat{\nabla} \theta_h}{\mathbf{B} \cdot \hat{\nabla} \theta} \quad \text{(F.12)}
\]
Lastly, the covariant vectors of the \((v, \zeta, \theta_h)\) coordinates are given by simple differentiation, and use of equation F.9:

\[
\vec{\nabla} v = \frac{2\pi}{B.\vec{\nabla} \theta_h} \vec{\nabla} \psi \tag{F.13}
\]

\[
\vec{\nabla} \zeta = \frac{\partial \zeta}{\partial \psi} \vec{\nabla} \psi + \frac{1}{2\pi} \vec{\nabla} \phi + \frac{F(\psi)}{2\pi} \left( \left\langle \frac{1}{R^2} \right\rangle - \frac{1}{R^2} \right) \vec{\nabla} \theta \tag{F.14}
\]

\[
\vec{\nabla} \theta_h = \frac{\partial \theta_h}{\partial \psi} \vec{\nabla} \psi + \vec{B}.\vec{\nabla} \theta_h \tag{F.15}
\]

With these remarks in mind, it is possible to easily prove properties 2 to 4.

By definition, \(\vec{B}.\vec{\nabla} v = 0\), and equation F.9 indicates that \(\vec{B}.\vec{\nabla} \theta_h\) is a flux function. Taking the scalar product of \(\vec{B}\) with equation F.14 results in:

\[
\vec{B}.\vec{\nabla} \zeta = \frac{1}{2\pi} \vec{B}.\vec{\nabla} \phi + \frac{F(\psi)}{2\pi} \frac{\vec{B}.\vec{\nabla} \theta}{\vec{B}.\vec{\nabla} \theta} \left( \left\langle \frac{1}{R^2} \right\rangle - \frac{1}{R^2} \right) \tag{F.16}
\]

Since by definition of \(F(\psi)\) and \(\phi\), \(\vec{B}.\vec{\nabla} \phi = F(\psi) R^{-2}\), we have:

\[
\vec{B}.\vec{\nabla} \zeta = \frac{F(\psi)}{2\pi} \left\langle \frac{1}{R^2} \right\rangle \tag{F.17}
\]

This equation guarantees that \(\vec{B}.\vec{\nabla} \zeta\) is the flux surface average of \(\vec{B}.\vec{\nabla} \phi\). Consequently, all contravariant components of the magnetic field are flux functions and property 3 is verified.

Substituting for \(\vec{B}.\vec{\nabla} \phi = R^{-2} F(\psi)\) in the safety factor definition gives:

\[
q(v) = \frac{1}{2\pi} \int \frac{F(\psi)/(2\pi R^2)}{\vec{B}.\vec{\nabla} \theta} d\theta \tag{F.18}
\]

\[
= \frac{F(\psi)}{2\pi} \int \frac{d\theta/R^2}{\vec{B}.\vec{\nabla} \theta} \tag{F.19}
\]

Now using the the definition of the flux surface average (equation 5.5) together with equations F.17 and F.9:

\[
q(v) = \left( \int \frac{d\theta}{\vec{B}.\vec{\nabla} \theta} \right) \vec{B}.\vec{\nabla} \zeta \tag{F.20}
\]

\[
= \frac{\vec{B}.\vec{\nabla} \zeta}{\vec{B}.\vec{\nabla} \theta_h} \tag{F.21}
\]

This demonstrates that property 2 is verified.

There only remains to prove property 4. This is done by calculating the Jacobian of the coordinates \((v, \zeta, \theta_h)\). Taking the cross-product of equations F.13 and
\[ \hat{\nabla} v \times \hat{\nabla} \zeta = \frac{1}{B \cdot \hat{\nabla} \theta_h} \left( \hat{\nabla} \psi \times \hat{\nabla} \phi + F(\psi) \left( \left\langle \frac{1}{R^2} \right\rangle - \frac{1}{R^2} \right) \frac{\hat{\nabla} \psi \times \hat{\nabla} \theta}{B \cdot \hat{\nabla} \theta} \right) \]  

(F.22)

Using equation F.12 to transform the second term on the right hand side:

\[ \hat{\nabla} v \times \hat{\nabla} \zeta = \frac{1}{B \cdot \hat{\nabla} \theta_h} \left( \hat{\nabla} \psi \times \hat{\nabla} \phi + F(\psi) \left( \left\langle \frac{1}{R^2} \right\rangle - \frac{1}{R^2} \right) \frac{\hat{\nabla} \psi \times \hat{\nabla} \theta_h}{B \cdot \hat{\nabla} \theta_h} \right) \]  

(F.23)

Taking the scalar product of equations F.23 with \( \hat{\nabla} \theta_h \), the second term on the right hand side vanishes leaving:

\[ \left( \hat{\nabla} v \times \hat{\nabla} \zeta \right) \cdot \hat{\nabla} \theta_h = \frac{1}{B \cdot \hat{\nabla} \theta_h} \left( \left( \hat{\nabla} \psi \times \hat{\nabla} \phi \right) \cdot \hat{\nabla} \theta_h \right) \]  

(F.24)

Substituting for \( \hat{\nabla} \theta_h \) on the right hand side using equation F.15 then yields:

\[ \left( \hat{\nabla} v \times \hat{\nabla} \zeta \right) \cdot \hat{\nabla} \theta_h = \frac{1}{B \cdot \hat{\nabla} \theta_h} \left( \left( \hat{\nabla} \psi \times \hat{\nabla} \phi \right) \left( \frac{\partial \theta_h}{\partial \psi} \hat{\nabla} \psi + B \cdot \hat{\nabla} \theta_h \frac{\hat{\nabla} \theta}{B \cdot \hat{\nabla} \theta} \right) \right) \]  

(F.25)

\[ = \left( \hat{\nabla} \psi \times \hat{\nabla} \phi \right) \cdot \frac{\hat{\nabla} \theta}{B \cdot \hat{\nabla} \theta} \]  

(F.26)

Using equations F.4 and F.26 eventually proves property 4:

\[ \left( \hat{\nabla} v \times \hat{\nabla} \zeta \right) \cdot \hat{\nabla} \theta_h = \frac{\left( \hat{\nabla} \psi \times \hat{\nabla} \phi \right) \cdot \hat{\nabla} \theta}{B \cdot \hat{\nabla} \theta} \]  

(F.27)

\[ = \frac{\tilde{B} \cdot \hat{\nabla} \theta}{B \cdot \hat{\nabla} \theta} \]  

(F.28)

\[ = 1 \]  

(F.29)

The \((v, \zeta, \theta_h)\) coordinates built using equations 5.2-5.4 therefore verify properties 1-4, and thus are the Hamada coordinates. Their construction is closely linked to the expression of the volume given by equation F.4 and the flux surface average defined in equation 5.5.
Appendix G

Calculation of \((\vec{\xi} \cdot \vec{\nabla}) \vec{B}\)

NTV theory requires the magnetic perturbation to be expressed in the Lagrangian form. The Lagrangian term to be added to the Eulerian form of \(\delta \vec{B}\) is simply the contribution of the displacement of the fluid cell to the perturbation, \((\vec{\xi} \cdot \vec{\nabla}) \vec{B} = \vec{B}_0(\vec{r} + \vec{\xi}) - \vec{B}_0(\vec{r}) + O\left(|\vec{\xi}|^2\right)\). To carry out this calculation, the equilibrium magnetic field is most conveniently decomposed on the usual cylindrical local basis \((\vec{e}_R, \vec{e}_\phi, \vec{e}_Z)\), but with the use of the \((\psi, \phi, \theta)\) coordinates to locate a point in space:

\[
\vec{B}_0(\vec{r}) = B_{0,R}(\psi, \theta) \vec{e}_R + B_{0,\phi}(\psi, \theta) \vec{e}_\phi + B_{0,Z}(\psi, \theta) \vec{e}_Z \quad (G.1)
\]

According to basic differential geometry, the equilibrium field at \(\vec{r} + \vec{\xi}\) is:

\[
\vec{B}_0(\vec{r} + \vec{\xi}) = \vec{B}_0(\vec{r}) + \frac{\partial \vec{B}_0}{\partial \psi} \left(\vec{\nabla}_\psi \cdot \vec{\xi}\right) + \frac{\partial \vec{B}_0}{\partial \phi} \left(\vec{\nabla}_\phi \cdot \vec{\xi}\right) + \frac{\partial \vec{B}_0}{\partial \theta} \left(\vec{\nabla}_\theta \cdot \vec{\xi}\right) + O\left(|\vec{\xi}|^2\right) \quad (G.2)
\]

Therefore:

\[
(\vec{\xi} \cdot \vec{\nabla}) \vec{B} = \frac{\partial \vec{B}_0}{\partial \psi} \vec{\xi}^\psi + \frac{\partial \vec{B}_0}{\partial \phi} \vec{\xi}^\phi + \frac{\partial \vec{B}_0}{\partial \theta} \vec{\xi}^\theta + O\left(|\vec{\xi}|^2\right) \quad (G.3)
\]

The differential forms of \(\vec{B}_0\) must be calculated taking into account that the field is axisymmetric and that the chosen basis vectors are not constant in space but depend on the toroidal angle. This gives, for \(\alpha \in (\psi, \theta)\):

\[
\frac{\partial \vec{B}_0}{\partial \alpha} = \frac{\partial B_{0,R}}{\partial \alpha} \vec{e}_R + \frac{\partial B_{0,\phi}}{\partial \alpha} \vec{e}_\phi + \frac{\partial B_{0,Z}}{\partial \alpha} \vec{e}_Z \quad (G.4)
\]
APPENDIX G. CALCULATION OF $(\vec{\xi}.\vec{\nabla})\vec{B}$

and:

$$\frac{\partial B_0}{\partial \phi} = B_{0,R} \frac{\partial \vec{e}_R}{\partial \phi} + B_{0,\phi} \frac{\partial \vec{e}_\phi}{\partial \phi} + B_{0,Z} \frac{\partial \vec{e}_Z}{\partial \phi} = B_{0,R} \vec{e}_\phi - B_{0,\phi} \vec{e}_R \quad \text{(G.5)}$$

Equations G.3, G.4, G.5 allow a simple calculation of the Lagrangian term of the magnetic perturbation, avoiding the intricate, although widespread, use of the $\vec{\nabla} \times (\vec{\xi} \times \vec{B})$ and $\vec{\nabla} (\vec{\xi} \vec{B})$ operators. Each of these individually diverge, but their diverging parts cancel each other in the combination involved in $(\vec{\xi}.\vec{\nabla})\vec{B}$. The Lagrangian term is given in its simple form by:

$$\left(\vec{\xi}.\vec{\nabla}\right)\vec{B} = \xi_\psi \left( \frac{\partial B_{0,R}}{\partial \psi} \vec{e}_R + \frac{\partial B_{0,\phi}}{\partial \psi} \vec{e}_\phi + \frac{\partial B_{0,Z}}{\partial \psi} \vec{e}_Z \right)$$

$$+ \xi_\phi \left( B_{0,R} \vec{e}_\phi - B_{0,\phi} \vec{e}_R \right)$$

$$+ \xi_\theta \left( \frac{\partial B_{0,R}}{\partial \theta} \vec{e}_R + \frac{\partial B_{0,\phi}}{\partial \theta} \vec{e}_\phi + \frac{\partial B_{0,Z}}{\partial \theta} \vec{e}_Z \right) \quad \text{(G.6)}$$

$$+ O \left( \left| \vec{\xi} \right|^2 \right)$$
Appendix H

Additional information

Information on the routines designed to carry out the analysis presented in this thesis, along with instruction on how to use them, are available on the Culham intranet:

The author’s permanent email address is: minh-duc.hua@polytechnique.org
Glossary

I. Greek Symbols

$\beta$  
Ratio of the kinetic to magnetic pressure

$\gamma_c$  
Ratio of specific heats for the single fluid

$\gamma$  
Growth rate of an MHD mode

$\vec{\Gamma}_{ld}$  
Flux of angular momentum

$\delta_s$  
Plasma skin depth

$\delta_w$  
Skin depth of the resistive wall fitted to stabilise the RWM

$\delta W \left( \vec{\xi}, \vec{\xi}^* \right)$  
Potential energy of the displacement $\vec{\xi}$ in the variational formulation of ideal linear MHD analysis

$\Delta q$  
Difference between the minimal $q$ value and 1, $q_{\text{min}} - 1$

$\Delta q_{\text{crit}}$  
Critical value of $\Delta q$ under which the LLM is destabilised

$\Delta w$  
Thickness of the resistive wall fitted to stabilise the RWM

$\epsilon$  
Local aspect ratio

$\epsilon_0$  
Vacuum permittivity

$\epsilon_p$  
Aspect ratio of the whole plasma

$\epsilon_{\text{SXR}}$  
SXR emissivity of the plasma

$\phi$  
Toroidal geometrical angle

$\Phi$  
Electrostatic potential

$\kappa$  
Plasma elongation

$\lambda_D$  
Debye length

$\mu$  
Kinematic viscosity

$\mu_0$  
Vacuum permeability

$\nu_*$  
Ion-ion collision frequency normalised to the thermal ion bounce time

$\nu_{ii}$  
Ion-ion collision frequency

$\omega$  
Toroidal angular frequency (see table III for possible species subscripts, no subscript implies single fluid toroidal angular frequency)

$\omega_0$  
Central angular frequency

$\omega_A$  
Alfvén frequency

$\omega_{b,i}$  
Thermal ion bounce frequency

$\omega_{t,i}$  
Thermal ion transit frequency
GLOSSARY

\[ \omega_{MHD} \] Angular frequency of the magnetic perturbation structure in presence of an MHD mode

\[ \omega_{NC}^{\ast} \] Offset angular frequency in the NTV theory

\[ \Omega \] Complex growth rate of an MHD mode

\[ \Omega^2 (\xi, \xi^\ast) \] Functional used in the variational formulation of ideal linear MHD analysis

\[ \omega_{L,\alpha} \] Larmor pulsation of a plasma species (see table III for possible species subscripts)

\[ \pi_{\alpha} \] Viscosity tensor (see table III for possible species subscripts, no subscript implies single fluid viscosity tensor)

\[ \psi \] Poloidal flux per unit toroidal angle

\[ \psi_N \] Normalised poloidal flux, \( \psi_N = \frac{\psi - \psi_{axis}}{\psi_{boundary} - \psi_{axis}} \)

\[ \rho \] Single fluid mass density

\[ \rho_* \] Ion Larmor radius normalised to the plasma minor radius

\[ \rho_{L,\alpha} \] Larmor orbit of a plasma species (see table III for possible species subscripts)

\[ \rho_c \] Charge density

\[ \sigma_{CX,\alpha\beta} \] Cross section of the charge exchange reaction between species \( \alpha \) and \( \beta \)

\[ \sigma_w \] Conductivity of the resistive wall fitted to stabilise the RWM

\[ \theta \] Poloidal geometrical angle

\[ \theta_h \] Hamada poloidal coordinate

\[ \theta_s \] Poloidal angle of the straight field line coordinates

\[ \tau_d \] Dissipation timescale in turbulence theory

\[ \tau_E \] Energy confinement time

\[ \tau_{eddy} \] Eddy turnover time in turbulence theory

\[ \tau_{shear} \] Modified eddy turnover time due to flow shear in turbulence theory

\[ \tau_{\alpha\beta} \] Collision time of species \( \alpha \) on species \( \beta \) (see table III for possible species subscripts)

\[ \tau_\phi \] Momentum confinement time

\[ \tau_w \] Resistive timescale of the resistive wall fitted to stabilise the RWM

\[ \chi_N^2 \] Normalised \( \chi^2 \) of a fit

\[ \chi_\phi \] Effective toroidal angular momentum diffusivity

\[ \chi_i \] Effective ion energy diffusivity

\[ \tilde{\xi} \] Displacement due to an MHD instability

\[ \zeta \] Hamada toroidal coordinate

II. Roman Symbols

\[ a \] Tokamak minor radius

\[ A(\psi) \] Area of the flux tube

\[ \bar{A} \] Potential vector of the electromagnetic field
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{B}$</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>$B_{\phi,0}$</td>
<td>Central magnetic field</td>
</tr>
<tr>
<td>$B_p$</td>
<td>Poloidal magnetic field</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$D_f$</td>
<td>Anomalous fast ion diffusivity</td>
</tr>
<tr>
<td>$e$</td>
<td>Elementary charge</td>
</tr>
<tr>
<td>$\vec{E}$</td>
<td>Electric field</td>
</tr>
<tr>
<td>$E_{NBI}$</td>
<td>Acceleration energy of the NBI heating system</td>
</tr>
<tr>
<td>$f_\alpha$</td>
<td>Distribution functions of a plasma species (see table III for possible species subscripts)</td>
</tr>
<tr>
<td>$\vec{f}_{ext}$</td>
<td>Forces external to the fluids other than electromagnetic per unit volume</td>
</tr>
<tr>
<td>$F(\psi)$</td>
<td>Poloidal current per unit toroidal angle</td>
</tr>
<tr>
<td>$\vec{F}(\vec{\psi})$</td>
<td>Force operator associated with and MHD induced displacement</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Plasma current</td>
</tr>
<tr>
<td>$I_{pol}$</td>
<td>Poloidal current</td>
</tr>
<tr>
<td>$J$</td>
<td>Moment of inertia</td>
</tr>
<tr>
<td>$\mathcal{J}$</td>
<td>Jacobian of a coordinate system</td>
</tr>
<tr>
<td>$\vec{\mathcal{J}}$</td>
<td>Current density vector</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$K(\vec{\xi},\vec{\xi}^*)$</td>
<td>Kinetic energy of the displacement $\vec{\xi}$ in the variational formulation of ideal linear MHD analysis</td>
</tr>
<tr>
<td>$l_{cor}$</td>
<td>Eddy coherence length in turbulence theory</td>
</tr>
<tr>
<td>$l_\phi$</td>
<td>Angular momentum density</td>
</tr>
<tr>
<td>$L_\phi$</td>
<td>Total angular momentum</td>
</tr>
<tr>
<td>$M_A$</td>
<td>Alfvén Mach number</td>
</tr>
<tr>
<td>$M_{th}$</td>
<td>Thermal Mach number</td>
</tr>
<tr>
<td>$m$</td>
<td>Poloidal mode number of an MHD instability</td>
</tr>
<tr>
<td>$m_\alpha$</td>
<td>Mass of a plasma species (see table III for possible species subscripts)</td>
</tr>
<tr>
<td>$n$</td>
<td>Toroidal mode number of an MHD instability (despite an overlap with the single fluid density notation, the confusion is unlikely to happen, this notation is therefore kept in order to comply with standard notations)</td>
</tr>
<tr>
<td>$n_\alpha$</td>
<td>Number density (see table III for possible species subscripts, no subscript implies single fluid number density)</td>
</tr>
<tr>
<td>$n_{el}$</td>
<td>Electron line-integrated density</td>
</tr>
<tr>
<td>$N_D$</td>
<td>Plasma parameter</td>
</tr>
<tr>
<td>$p_\alpha$</td>
<td>Scalar pressure (see table III for possible species subscripts, no subscript implies single fluid number density)</td>
</tr>
<tr>
<td>$p_\phi$</td>
<td>Canonical angular momentum</td>
</tr>
</tbody>
</table>
GLOSSARY

\( p_{MA} \) Peaking factor of the Alfvén Mach number
\( p_{Mth} \) Peaking factor of the Thermal Mach number
\( p_{NBI} \) Linear momentum of the NBI particles
\( P_{in} \) Auxiliary heating power
\( P_{NBI} \) NBI heating power
\( \vec{r} \) Space coordinate
\( \mathbf{R}_{\beta \neq \alpha} \) Force interaction with other fluid species in the multiple fluid descriptions
\( r \) Geometrical minor radius
\( R_0 \) Plasma major radius
\( R^2 \) Pearson coefficient of determination of a fit
\( R_{mid} \) Geometrical major radius of outboard midplane of the considered flux surface
\( R_{NBI} \) Injection tangency radius of the NBI system
\( r_w \) Minor radius of the resistive wall fitted to stabilise the RWM
\( q \) Safety factor
\( q_0 \) Central safety factor
\( q_{95} \) Safety factor at \( \psi_N = 0.95 \)
\( q_\alpha \) Charge of a plasma species (see table III for possible species subscripts)
\( T \) Temperature (see table III for possible species subscripts, no subscript implies single fluid number density)
\( t_{\phi} \) Torque density
\( T_{\phi} \) Total torque
\( V \) Plasma volume
\( \vec{v} \) Fluid velocity (see table III for possible component subscript and species subscripts, no species subscript implies single fluid velocity)
\( \vec{v}_p \) Particle velocity (see table III for possible component subscript and species subscripts)
\( \vec{v} \) Velocity as a coordinate of velocity space
\( \vec{\tilde{v}} \) Particular velocity (deviation of the velocity of a plasma particle to the single fluid velocity)
\( \vec{v}_p \) Momentum pinch velocity
\( v_{th,\alpha} \) Thermal velocity of a plasma species (see table III for possible species subscripts)
\( W_{kin} \) Plasma thermal energy
\( Z_\alpha \) Charge of a plasma fluid species (see table III for possible species subscripts)
\( Z_{eff} \) Effective charge of the ion fluid
III. Subscripts and superscripts

- $\alpha$: Generic subscript for particle species or fluid species
- $\psi_N$: Component relative to the normalised flux
- $\phi$: Toroidal component
- $\theta$: Poloidal component
- $\theta_s$: Poloidal component in the straight field line coordinates
- $COV$: Covariant component of a vector
- $CON$: Contravariant component of a vector
- $e$: Electron species
- $f$: Fast ion species
- $i$: Ion species
- $NBI$: Fast ion arising from NBI
- $r$: Radial component in the toroidal coordinate system
- $R$: Radial component in the cylindrical coordinate system
- $Z$: Altitude component in the cylindrical coordinate system
- $\parallel$: Component parallel to the $\vec{B}$ field
- $\perp$: Component perpendicular to the $\vec{B}$ field

IV. Brackets and overscripts

- $X^*$: Complex conjugate of $X$
- $\langle X \rangle$: Flux surface average of $X$
- $\langle X \rangle_p$: Profile average of $X$
- $\langle X \rangle_v$: Volume average of $X$

V. Acronyms and Names

- AT: Advanced Tokamak
- CXRS: Charge Exchange Recombination Spectroscopy
- EFIT: Equilibrium Fitting Code
- ELM: Edge Localised Mode
- FWHM: Full Width Half Maximum
- HDI: Human Development Index
- HELENA: Fixed boundary static equilibrium reconstruction code
- ICRH: ICRH Cyclotron Resonance Heating
- ITB: Internal Transport Barrier
- ITER: International Thermonuclear Experimental Reactor
- IEA: International Energy Agency
- JET: Joint European Torus
- LLM: Long-Lived Mode
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>Ordinary Least Squares fitting</td>
</tr>
<tr>
<td>MAST</td>
<td>Mega Ampère Spherical Tokamak</td>
</tr>
<tr>
<td>MHD</td>
<td>MagnetoHydroDynamics</td>
</tr>
<tr>
<td>MSE</td>
<td>Motional Stark Effect measurement system</td>
</tr>
<tr>
<td>NBI</td>
<td>Neutral Beam Injection</td>
</tr>
<tr>
<td>NTM</td>
<td>Neoclassical Tearing Mode</td>
</tr>
<tr>
<td>NTV</td>
<td>Neoclassical Toroidal Viscosity</td>
</tr>
<tr>
<td>RWM</td>
<td>Resistive Wall Mode</td>
</tr>
<tr>
<td>ST</td>
<td>Spherical Tokamak</td>
</tr>
<tr>
<td>SXR</td>
<td>Soft X-Ray</td>
</tr>
<tr>
<td>TS</td>
<td>Thomson Scattering measurement system</td>
</tr>
<tr>
<td>TRANSP</td>
<td>Monte-Carlo TRANSPort Code</td>
</tr>
</tbody>
</table>
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Related publications


M-D Hua, IT Chapman, SD Pinches, RJ Hastie, and the MAST team, submitted to *Physical Review Letters*, “Saturated internal instabilities in advanced tokamak plasmas”

Scaling of rotation and momentum confinement in JET plasmas

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Abstract
An extensive database to study the scaling of rotation and momentum transport has been constructed at JET. The database contains information from various operational scenarios, amongst them H-mode discharges, and parameters that characterize the rotation, as well as those that describe the general plasma conditions. JET plasmas are predominantly heated by neutral beam injection which is also the main source for the observed toroidal rotation. Dimensionless Mach numbers are introduced to quantify rotation. The scaling of plasma rotation and the Mach numbers in particular has been studied. The thermal and Alfvén Mach numbers were found to scale inversely with q and with the ratio of torque and additional heating power. Although the momentum and energy confinement times were found to be of the same magnitude, the ratio was found to vary. Regression analyses showed a dependence of both the energy and momentum confinement times on plasma rotation. If rotation was included in the scaling model of energy and momentum confinement the quality of the fits substantially improved. Detailed analysis of the core and edge (pedestal) confinement showed that momentum confinement was improved in the core of the plasma compared with the energy confinement. However, the pedestal proved to be less confining for the momentum than for the energy.

PACS numbers: 52.55.Fa, 66.20.+d

1. Introduction
Rotation of tokamak plasmas is thought to play an important role in plasma stability and the suppression of turbulence [1,2]. It is therefore important to understand the scaling of plasma rotation and momentum confinement, in order to accurately predict ITER performance.

Previous analysis had shown a relationship between energy and momentum confinement [3–7]. It is often reported that these quantities have comparable magnitudes and have similar dependences with individual plasma parameters. This is usually attributed to the coupling of heat diffusivity and viscosity. However, detailed studies have also shown cases where this relationship breaks down. In ASDEX a slightly different scaling was found for the momentum and energy confinement time, while a strong deviation was observed in discharges with peaked density profiles [8]. Similarly, a significant difference between momentum and energy confinement was found in JET discharges during ELM free phases [9]. Detailed analysis has shown that the momentum confinement time in JET plasmas does not necessarily equal the energy confinement time and that the momentum and heat diffusivities can differ significantly in the core [10]. Recent experiments at DIII-D used combined co- and counter-current neutral beam injection (NBI) in order to produce plasmas with varying net torque. The experiments showed a dependence on the plasma performance with the net applied torque [11]. As plasma rotation is expected to affect turbulence, this raises the question of whether the energy and momentum confinement may depend on the rotation.
At JET, a database has been created to study the general scaling of plasma rotation and momentum confinement in order to get a better understanding of the relevant parameter dependences. The database contains information on various operational scenarios such as the ELMy H-mode baseline scenario, hybrid and advanced tokamak scenarios. Usually the dominant auxiliary heating system in these JET scenarios is NBI, which also supplies a considerable toroidal torque to the plasma, hence driving large toroidal rotation. Toroidal angular rotation frequencies up to $\omega_{\phi} = 222 \, \text{krad s}^{-1}$ have been observed in JET, which is equivalent to rotation velocities of almost $700 \, \text{km s}^{-1}$. However, for ITER plasmas, with larger inertia and lower available NBI torque, plasma rotation is expected to be considerably lower. Therefore, a large number of discharges with dominant ion cyclotron resonance heating (ICRH), which may be representative of low torque plasmas, have also been added to the database.

Nevertheless, because of the dominant and uni-directional NBI system at JET, there is a finite external toroidal torque for all database entries. Intrinsic or spontaneous rotation has been observed in various devices without net external momentum input. For example in purely ICRH-heated plasmas [12] or those heated by NBI but with a balanced torque [13]. Such balanced NBI operation is not possible at JET. A detailed analysis of the scaling of intrinsic rotation has been presented in [14]. This paper does not intend to analyse intrinsic or ICRH driven rotation in JET and details of such studies can be found elsewhere [15, 16]. The aim here is to describe the general magnitude and scaling of NBI driven rotation and momentum confinement in JET.

Plasma rotation in JET is measured by means of charge exchange recombination spectroscopy (CXRS) [5]. It determines both the toroidal plasma rotation and ion temperature profile at 12 radial locations. The measured quantities are those of carbon ions and in this study it is assumed that the main plasma ions have equal temperature and velocities. For H-mode plasmas this assumption generally holds. Although for plasmas with large pressure gradients, such as those with an internal transport barrier (ITB), the rotation profiles may need to be corrected [17]. Typical corrections for plasmas with an ITB are $\Delta v/v_c \sim 25-35\%$, where $v_c$ is the measured carbon velocity. For H-mode discharges the correction never exceeds $\Delta v/v_c < 5\%$ which is within the accuracy of the diagnostic. The effect of this correction for global parameters, such as total angular momentum, which are derived from integrated or averaged profiles, is however often within the errors of these parameters.

The paper first describes the JET rotation database in section 2, listing the parameters and discussing the data selection and validation. Although, a large relational database has the advantage of showing overall trends and scaling, detailed variations may often be hidden by data scatter. Hence, proper data validation is required together with an understanding of the errors and parameter correlations, which is treated in this section.

Section 3 presents the scaling of the thermal and Alfvén Mach numbers in JET plasmas. These are, respectively, defined as the ratio of rotation velocity and the thermal or Alfvén velocity. These are dimensionless parameters, which enable a straightforward comparison between various operational scenarios at JET or even with other devices. General profile information is also included in the database and the variation of the Mach number and toroidal rotation profiles for the different scenarios are discussed. This section also shows the general scaling of both Mach numbers in JET.

The global momentum confinement time is discussed in section 4, where it is compared with the total energy confinement time. Furthermore, differences between confinement of momentum in the core and that by the edge pedestal are studied. A regression analysis has been carried out to determine the principal parameter dependences for momentum confinement. Some detailed cases are discussed to highlight special dependences. The results are summarized and discussed in the final section of this paper.

2. Rotation database at JET

In this section the details of the JET rotation and momentum transport database are presented. Identifying broad trends in plasma rotation and the global scaling of plasma parameters could be achieved with a relational database. The latter must include a large number of discharges from all operation modes, with the aim of avoiding hidden correlations between parameters. Particular effort was dedicated to finding a good compromise between computing time and accuracy of the data. In addition, the database was designed to be easily generated, updated and used in any data analysis software.

2.1. Database entries and parameters

As mentioned above, the database entries include a range of operation scenarios; the baseline ELMy H-mode scenario, the advanced tokamak scenarios with ITBs and the so-called hybrid scenario. These database entries are a subset of dedicated databases for each of these operation scenarios, such as the main JET H-mode confinement database [18] and those for the ITB [19] and hybrid scenario [20]. These scenarios are predominantly heated by NBI, but a selected group of discharges with a dominant fraction of ICRH has also been added. These entries are taken mainly from experiments that studied ICRH driven plasma rotation [15,16]. Furthermore, the database contains an additional subset of JET discharges from experiments with reversed plasma current and toroidal field direction, i.e. counter-current NBI. Most of these additional entries are H-modes. Approximately, 80% of all database entries can be described as H-mode, showing a characteristic edge pedestal and ELMs. The scenarios that appear in the rotation database are summarized in table 1, the total number of database entries for the version used in this paper (May 2007) is 574.

All discharges are analysed in a steady-state phase. The used data are reliable in a sense that these consist of subsets of other existing databases or concerns discharges that have appeared in previous publications. To reduce parameter errors, each data signal is averaged over a 200 ms steady-state time-window. The database parameters falls into four categories: general, energy, rotation and profile parameters. In table 2, a summary is given of the main database parameters for each category. The values for $\gamma$, $\beta$ and $\rho^*$ have been determined from global parameters as shown in [18]. Note that besides global values also data describing the H-mode
pedestal energy and momentum have been added, which are analysed in section 4. In addition the database contains all information on data computation procedures, estimated error bars and possible computing errors that have occurred. When available, additional information on the background of the pulses, analysis carried out and comments of previous users is provided.

Besides the parameters mentioned in table 2, other parameters are included in the database, for instance: normalized gradient lengths of ion temperature and rotation parameters, thermal and momentum diffusivities or parameters characterizing the ITB. These have been omitted in table 2, because they are not used in the analysis presented in this paper.

2.2. Errors, validation and correlation

In order to carry out a reliable database analysis, parameters should be validated and a proper understanding of the data errors (see table 2), and data scatter is required. Furthermore, not all parameters in the database may be entirely independent and correlations may complicate regression analysis.

The database was designed to carry out studies involving numerous discharges over several operation scenarios. To achieve this, short computing time calculations are automatically performed using JET’s diagnostics raw signals. Some parameters, such as the normalized collisionality, cannot be reasonably computed directly. In this case, an approximated formula is used.

For each parameter of the database, a single error accounts for the uncertainty on the value of all entries. It does not take into account individual discharge conditions which might influence the data scatter. Some of the parameters can have a large uncertainty, for example the effective ion charge, \(Z_{\text{eff}}\). It is determined from the impurity densities measured by CXRS and may be underestimated because not all impurities are included. For parameters computed from an approximated formula, the error bars are derived from benchmarks against reliable data. The parameters involving the calculation of a gradient have large error bars, of the order of 40%. Existence diagrams of various parameters are shown in figure 1.

The size of the database implies that the accuracy of every single data point cannot be individually guaranteed. Nevertheless all orders of magnitude are checked as well as consistency with basic or well-known scalings, as for instance the correlation of torque and angular momentum (see figure 1(a)). For the counter-current data, the absolute value of the torque and rotation is shown, which is done throughout this paper. The toroidal torque, as used in this paper, is that supplied by the tangential NBI system of JET and no other sources of momentum have been considered. The data scatter observed in this graph is expected due to differences in the momentum confinement as discussed later in this paper. Depending on how they were calculated, the database parameters are validated by different means. For parameters characterizing the overall discharge such as line integrated density, the heating powers or the plasma current, a basic hand-check is done during the time-window determination. An in-depth look at all parameters for every outlier observed while using the database is also taken. As noted before, parameters computed by means of a more complex calculation, e.g. gradient computation or use of a scaling law, are benchmarked by comparison with available results from other computation methods or databases.

In contrast to the scan of a given parameter keeping all other parameters constant, the entire parameter set usually varies from one database discharge to another. Database discharges were performed over a large time span, meaning that the wall conditioning, or even the physical structure of JET, which underwent several divertor changes, could have changed. These features are sources of data scatter and may confuse the analysis.

The correlation between parameters affects the database regression analysis. In table 3 the correlations between the logarithms of a number of relevant parameters are shown. Characterizing the database discharge, for example as line integrated density, the heating powers or the plasma current, a basic hand-check is done during the time-window determination. An in-depth look at all parameters for every outlier observed while using the database is also taken. As noted before, parameters computed by means of a more complex calculation, e.g. gradient computation or use of a scaling law, are benchmarked by comparison with available results from other computation methods or databases.

3. Mach number scaling in JET plasmas

The JET rotation database has been used to study the characteristics and scaling of plasma rotation, which will be presented in this section. Using dimensionless parameters is beneficial when studying the scaling of rotation, as it allows a straightforward comparison between various JET scenarios.
or even other devices. Previously the thermal Mach number was introduced [10], defined as the ratio of the kinetic and the thermal velocity:

\[ M_{th} = \frac{v_{kin}}{v_{th}} = \sqrt{\frac{m}{\epsilon}} \frac{v_{th}}{T}. \]  

(1)

Here, \( m \) is the mass of the species and \( v_{th} \) its (toroidal) rotation velocity in (m s\(^{-1}\)). The temperature, \( T \), is given in (eV) with \( \epsilon \) the electron charge. The Mach number is the square root of the ratio of the kinetic energy and thermal energy. It depends both on the ratio of torque and heating power and the ratio of heat and momentum confinement. The Mach numbers given in this paper are for deuterium fluid. Furthermore, one could define the Alfvén Mach number as the ratio between the plasma rotation and the Alfvén velocities:

\[ M_A = \frac{v_{kin}}{v_A} = \frac{v_{th}}{B_0/\sqrt{\mu_0 \rho}}. \]  

(2)

Here, \( B_0 \) is the (toroidal) magnetic field strength in (T) and \( \rho \) is the mass density of the plasma species in (kg m\(^{-3}\)). The Alfvén Mach number is related to the electromagnetic properties of the plasma and thus relevant to physics such as resistive wall mode stability [21]. In contrast, the thermal Mach number is connected to instabilities or turbulences arising from fluid physics [22, 23]. It is interesting to note that the ratio of the Alfvén and thermal Mach number, as defined above, can be rewritten as

\[ \left( \frac{M_A}{M_{th}} \right)^2 = \frac{1/2n\epsilon T}{B_0^2/2\mu_0} \approx \frac{1}{3} \beta_\phi, \]  

(3)

where \( \beta_\phi \) is kinetic pressure of the plasma normalized to the (toroidal) magnetic pressure. Here \( n \) is the particle density of the plasma species and \( T \) is again in (eV). Because \( \beta_\phi \) is of the order of a few per cent, the Alfvén Mach number is expected to be approximately one order of magnitude smaller than the thermal Mach number. The thermal Mach number has a dependence on the temperature (see equation (1)), while the Alfvén Mach number relates to the local density (or \( \rho \) in equation (2)). In this section the general scaling of both these parameters is studied and their values for various JET scenarios are discussed.

### 3.1. Thermal Mach number

The relationship between the rotation and temperature profiles in predominantly NBI heated plasmas has previously been reported [6, 10]. These studies revealed an interesting trend between the central rotation velocity and ion temperature. An

---

**Table 2. An overview of the main database parameters.** For several parameters the range and an indication of the approximate error are given.

<table>
<thead>
<tr>
<th>Category</th>
<th>Name</th>
<th>Symbol</th>
<th>Unit</th>
<th>Range</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>Central electron line-integrated density</td>
<td>( n_e )</td>
<td>( 10^{20} \text{ m}^{-2} )</td>
<td>3.90–28.2</td>
<td>1%</td>
</tr>
<tr>
<td></td>
<td>Central toroidal magnetic field</td>
<td>( B_\phi )</td>
<td>T</td>
<td>0.99–3.56</td>
<td>2%</td>
</tr>
<tr>
<td>Plasma current</td>
<td>( I_p )</td>
<td>MA</td>
<td>0.98–3.99</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>Toroidal ( \beta ) (normalized pressure)</td>
<td>( \beta_\phi )</td>
<td>%</td>
<td>0.17–3.00</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Normalized ( \beta )</td>
<td>( \beta_n )</td>
<td>%Tm MA(^{-1} )</td>
<td>0.32–2.99</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Normalized collisionality</td>
<td>( \nu^* )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Normalized Larmor Radius</td>
<td>( \rho^* )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Average ion effective charge</td>
<td>( Z_{eff} )</td>
<td>e</td>
<td>1.00–4.20</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>NBI input power</td>
<td>( P_{NBI} )</td>
<td>MW</td>
<td>1.85–21.0</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>ICRH input power</td>
<td>( P_{CRH} )</td>
<td>MW</td>
<td>0.74–7.45</td>
<td>2%</td>
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<tr>
<td></td>
<td>LHCD input power</td>
<td>( P_{LHCD} )</td>
<td>MW</td>
<td>0.20–2.85</td>
<td>5%</td>
</tr>
<tr>
<td>Central ion temperature</td>
<td>( T_i(0) )</td>
<td>keV</td>
<td>5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central electron temperature</td>
<td>( T_e(0) )</td>
<td>keV</td>
<td>5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diagnostics energy</td>
<td>( W_{di} )</td>
<td>MJ</td>
<td>0.68–12.2</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Total kinetic energy</td>
<td>( W_{kin} )</td>
<td>MJ</td>
<td>0.51–9.09</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Pedestal kinetic energy</td>
<td>( W_{ped} )</td>
<td>MJ</td>
<td>0.22–5.64</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Total energy confinement time</td>
<td>( \tau_L )</td>
<td>s</td>
<td>5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pedestal energy confinement time</td>
<td>( \tau_{ped} )</td>
<td>s</td>
<td>15%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kinetic energy confinement time</td>
<td>( \tau_{ke} )</td>
<td>s</td>
<td>0.067–0.493</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>Rotation</td>
<td>Central angular frequency</td>
<td>( \omega(0) )</td>
<td>kHz s(^{-1} )</td>
<td>2.46–222</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>Central ion toroidal velocity</td>
<td>( v_{th}(0) )</td>
<td>km s(^{-1} )</td>
<td>8.72–687</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>Maximal ion toroidal velocity</td>
<td>( v_{th,max} )</td>
<td>km s(^{-1} )</td>
<td>8.72–706</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>Total toroidal angular momentum</td>
<td>( L_\phi )</td>
<td>kg m(^2) s(^{-1} )</td>
<td>0.04–9.04</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>Pedestal toroidal angular momentum</td>
<td>( L_{ped} )</td>
<td>kg m(^2) s(^{-1} )</td>
<td>0.056–4.21</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>Toroidal torque (by NBI)</td>
<td>( T_\phi )</td>
<td>N m</td>
<td>0.31–23.1</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>Total momentum confinement time</td>
<td>( \tau_\phi )</td>
<td>s</td>
<td>0.049–0.585</td>
<td>8%</td>
</tr>
<tr>
<td></td>
<td>Pedestal mom. confinement time</td>
<td>( \tau_{ped} )</td>
<td>s</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Central thermal Mach number</td>
<td>( M_{th}(0) )</td>
<td>—</td>
<td>0.03–0.76</td>
<td>16%</td>
</tr>
<tr>
<td></td>
<td>Averaged thermal Mach number</td>
<td>( \langle M_{th} \rangle )</td>
<td>—</td>
<td>0.02–0.62</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>Central Alfvén Mach number</td>
<td>( M_A(0) )</td>
<td>—</td>
<td>0.001–0.05</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>Averaged Alfvén Mach number</td>
<td>( \langle M_A \rangle )</td>
<td>—</td>
<td>0.0000–0.07</td>
<td>6%</td>
</tr>
<tr>
<td>Profile</td>
<td>Thermal Mach number peaking factor</td>
<td>( p_{Mth} )</td>
<td>—</td>
<td>17%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Alfvén Mach number peaking factor</td>
<td>( p_{Ma} )</td>
<td>—</td>
<td>21%</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1. (a) The total toroidal angular momentum as a function of the total toroidal torque for various scenarios. (b) The central toroidal magnetic field as a function of the plasma current. (c) The total toroidal torque as a function of the total input power. (d) The line-integrated electron density as a function of the NBI power input.

Related publications

offset linear scaling with the square root of the temperature is seen in figure 2(a). The thermal Mach number for these points is given by the slope of a line from that point through the origin. A similar trend as shown in figure 2(a) is found when the total angular momentum is plotted versus the plasma kinetic energy. The Mach number may vary locally depending on the rotation and temperature profiles. A global value can be derived by means of the profile average Mach number. In figure 2(b), this average thermal Mach number, \( \langle M_{th} \rangle \), is plotted as a function of the ratio of toroidal torque, \( T_\phi \), and total auxiliary power, \( P_{in} \). NBI is taken as the only source of toroidal torque. The average thermal Mach number is found to scale approximately with the ratio of torque to total heating power. The rotation and thermal velocity (in equation (1)) are determined by the sources (torque and power) and the losses determined by the characteristics of the momentum and energy confinement. If the momentum and energy confinement are linked one would expect the thermal Mach number to scale with the ratio of the sources, i.e. \( T_\phi / P_{in} \). Although the torque and total auxiliary power parameters in JET are correlated and hence their ratio has a restricted range, a significant scaling with the ratio of these parameters can still be observed in figure 1(c). The relation between momentum and energy confinement is discussed in detail in the next section.

For the main scenarios at JET (i.e. H-mode, hybrid, ITB), which are often predominantly NBI heated, the average thermal Mach number is found in a range between 0.2 < \( \langle M_{th} \rangle \) 0.5. However, considerably lower values are found for the discharges with predominantly ICRH heating. From those discharges with a torque less than \( T_\phi = 1 \text{ Nm} \) showed thermal Mach numbers in the range 0.034 < \( \langle M_{th} \rangle \) 0.14. The average values for each scenario are \( \langle M_{th} \rangle = 0.36 \pm 0.09 \) for type I ELM My H-mode, \( \langle M_{th} \rangle = 0.25 \pm 0.07 \) for type III ELM My H-mode, \( \langle M_{th} \rangle = 0.34 \pm 0.06 \) for the hybrid scenario, which feature predominantly type I ELMs and \( \langle M_{th} \rangle = 0.31 \pm 0.08 \) for discharges with ITBs. The values quoted here are in each case the mean of the database subsets and its standard deviation. A significantly lower thermal Mach numbers is found for discharges with type III ELMs.
In figure 2(c), the peaking factor of the thermal Mach number profile (defined as \(M_\text{th}(0)/(\langle M_\text{th} \rangle)\)) is shown as a function of density. If the peaking factor is unity the rotation profile shape equals that of the square root temperature profile. The Mach number profile is more peaked for low density, predominantly NBI heated, ITB and hybrid, discharges, with peaking factors up to \(M_\text{th}(0)/(\langle M_\text{th} \rangle) = 1.8\), while it is almost flat for high-density H-modes. The central Mach numbers in discharges with ITBs can reach peak values up to \(M_\text{th}(0) = 0.68\), as shown in figure 2(d). The carbon Mach numbers are a factor \(\sqrt{m_c/m_i}\approx 2.4\) larger than those of the deuterium fluid given in figure 2 and for many discharges in the database the carbon velocity is supersonic with \(\langle M_\text{th} \rangle > 1\). Hollow Mach profiles, i.e. \(M_\text{th}(0)/(\langle M_\text{th} \rangle) < 1\), are found for a number of discharges in the database (see figure 2(c)). Hollow thermal Mach profiles do not necessarily mean that the velocity profiles are hollow too. Either the ion temperature profile is strongly peaked while the rotation profile remains flat or the rotation profile itself is also hollow. Some entries with \(M_\text{th}(0)/(\langle M_\text{th} \rangle) < 1\) are found to be high-density and counter-current NBI heated discharges, which are characterized by flat rotation profiles. The majority of hollow Mach profiles, however, are found for discharges with dominant ICRH heating where one expect a similar decoupling of temperature and rotation profile. Furthermore, it has been shown that ICRH could drive off-axis momentum in JET, yielding hollow rotation profiles [15].

3.2. Alfvén Mach number

The Alfvén and thermal Mach number are related according to equation (3) and their ratio is \(\beta_\text{A}/\beta_\text{th}\). Hence, the Alfvén Mach number is an order of magnitude smaller than the thermal one. In figure 3(a) the squared ratio of the Alfvén and thermal Mach number is plotted against \(\beta_\text{A}\). The data follow the trend given by equation (3), however, the \(\beta_\text{A}\) values are slightly larger because these data are based on the diamagnetic energy and contains a fast-particle energy component, while the vertical axis is based on thermal energy only. A number of ITB entries show a notable deviation from this trend. This may be due to an underestimation of \(Z_{\text{eff}}\) for these entries, which is used to determine the ion mass density in equation (2).

In figure 3(b) it can also be seen that the profile average Alfvén Mach number \(\langle M_\text{A} \rangle\) scales approximately with \(\beta_\text{A}\). A similar dependence was found for Alfvén Mach numbers in non-NBI driven plasmas [14]. Nonetheless the trend shown here could also be due to the coupling between the heating and torque sources in these predominantly NBI driven JET discharges. A detailed look shows that the \(\langle M_\text{A} \rangle\) is also lower for type III ELMy mode discharges compared with those with type I ELMs. Note that the higher central and average values found for ITB discharges may be overestimated as discussed above. However, as we have seen with the thermal Mach number, higher core rotation values are observed in plasmas with ITBs.

The Alfvén Mach profile peaking factor is defined as \(p_\text{MA} = M_\text{A}(0)/(\langle M_\text{A} \rangle)\). When the definitions of both Mach numbers are compared (see equations (1) and (2)), it is clear that the Alfvén Mach number profile is expected to be more peaked than thermal Mach number profile. The \(M_\text{A}\) profile peaking is a combination of the rotation and density profile peaking. In general the profiles are more peaked for low-density discharges, as shown in figure 3(c). The observed inverse trend with the density can be explained by the fact that for higher density plasmas, the torque deposition will be more off-axis, as the NBI penetration is reduced. Hence the higher density results in flatter rotation and Mach number profiles. Furthermore, the Mach profile peaking may be related to a peaking of the density profile, as mentioned before.

In [21], the relevant Alfvén Mach number, which determines stability to resistive wall modes, is not that in the centre or the average value but that at the position of an outer rational \(q\)-surface. The large peaking factor of the Alfvén Mach profile implies that the Alfvén Mach numbers at the edge are considerably lower than the values of \(M_\text{A}(0)\) and \(\langle M_\text{A} \rangle\). Hollow Mach number profiles are found in discharges with mainly ICRH heating or those with counter NBI, as was also found for the thermal Mach profiles for these discharges. Discharges with dominant ICRH heating featured the lowest Alfvén Mach numbers. From those discharges with \(T_\phi < 1\) N m the Alfvén Mach numbers were in the range \(0.0009 < \langle M_\text{A} \rangle < 0.008\). It is interesting to note that Alfvén Mach numbers in JET with a significant NBI torque are of similar magnitude as those found in devices that do not apply external momentum sources [14].
A further observation was that type III ELMy H-mode plasmas have generally lower average Mach numbers compared with those with type I ELMs. It has been reported previously that the transition to H-mode increases the edge thermal Mach number [10]. H-mode discharges have a flatter thermal Mach profile. In figure 5 an example is shown with a spontaneous transition from type I to III ELMs at $t = 22.75$ s. Both central and edge thermal Mach numbers decrease, but the edge Mach number drops more rapidly. This results in a more peaked Mach profile after the transition to type III ELMs. The average Mach number decreases. The angular momentum drops by 40% while the total energy only decreases by 13%. Because the momentum and heat sources remain constant, this indicates a change in the ratio of momentum and energy confinement times.

The formation of an H-mode pressure pedestal also affects the edge rotation. A clear example is presented in figure 6 that shows a discharge which has a marginal H-mode. Phases with good confinement and clear type I ELMs are followed by short phases, which can be characterized by a high frequency compound ELMs, and lower momentum confinement. Note that the rise in toroidal angular momentum ($\sim 28\%$) at each transition is more pronounced than that of the plasma energy ($\sim 19\%$). The central thermal and Alfvén Mach number are hardly affected whereas both edge Mach numbers increase significantly. The increase in angular momentum can be attributed to the higher edge rotation and density in this phase. During the compound phase the edge Alfvén Mach number remains at a low level ($M_{\text{edge}}^A = 0.005$). But the pedestal formation shows a large rise in the edge Alfvén Mach number which can reach a value of $M_{\text{edge}}^A = 0.008$ just before an ELM. This increase in $M_{\text{edge}}^A$ is due in part to both a faster rotation velocity and an increase in edge density (see equation (2)). It was recently reported that toroidal rotation could influence the H-mode pedestal [24]. From the observations presented here, it remains however unclear if the changes in rotation are due to a difference in pedestal momentum confinement or that rotation affects the pedestal confinement itself.
3.4. Scaling of Mach numbers

The average thermal and Alfvén Mach numbers, \( \langle M_\text{th} \rangle \) and \( \langle M_A \rangle \), can be used as global parameters to characterize rotation. The total angular momentum is another option, although it is not dimensionless. In figures 2 and 3, the basic trends are shown. A regression analysis was carried out for a more detailed investigation of the main parameter scaling. Besides the scaling, the analysis also indicates the coupling between the various parameters. The quality of the fit of the model to the data is given by the Pearson correlation coefficient, which is unity for a perfect fit. The normalized \( \chi^2 \), which is defined as the co-variance between model and measurements normalized to the measurement error, is also an indicator to the quality of the fit.

The model that has been used provides a non-linear scaling of the Mach numbers with the main engineering parameters that define a tokamak discharge: the line-integrated density, \( n_e \), plasma current, \( I_p \), toroidal magnetic field, \( B_T \), toroidal torque, \( T_\phi \), and total heating power, \( P_{\text{th}} \). The regression analysis is carried out by a linear fit to the logarithm of the non-linear model. A model with a reduced set of parameters showed a degradation of the fit result, while adding other parameters, like \( Z_{\text{eff}} \), did not significantly improve the fit, suggesting that the above model uses the optimum parameter set. The results for \( \langle M_\text{th} \rangle \) and \( \langle M_A \rangle \) are, respectively,

\[
\langle M_\text{th} \rangle \propto n_e^{-0.12\pm0.03} \cdot I_p^{0.49\pm0.06} \cdot B_T^{-0.43\pm0.08} \cdot T_\phi^{0.73\pm0.02} \cdot P_{\text{th}}^{-0.51\pm0.03}.
\]

(4)

\[
\langle M_A \rangle \propto n_e^{-0.08\pm0.04} \cdot I_p^{0.80\pm0.08} \cdot B_T^{1.12\pm0.12} \cdot T_\phi^{-0.95\pm0.04} \cdot P_{\text{th}}^{0.36\pm0.04}.
\]

(5)

The Pearson correlation coefficients and \( \chi^2 \) for both fits (4) and (5) were found to be \( \rho = 0.88 \), \( \chi^2 = 1.11 \) and \( \rho = 0.84 \), \( \chi^2 = 1.14 \), respectively. The errors for each coefficient relate to the standard deviation within which the regression analysis would produce a fit of equal quality. In order to get a better feeling for the quality of the scaling, their respective values are plotted versus the measured ones in figures 7(a) and (b). The analysis was carried out using the complete database and the coupling between the logarithms of the parameters in this model can be found in table 3(a). As also seen from

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{(a) The squared ratio of the Alfvén and thermal Mach number plotted versus \( \bar{\beta}_e \). (b) The profile averaged Alfvén Mach number, \( \langle M_A \rangle \), as a function of \( \bar{\beta}_e \). (c) The peaking factor of the Alfvén Mach profile, \( M_A(0)/\langle M_A \rangle \), as a function of the line-averaged density. (d) The Alfvén Mach number in the centre of the plasma, \( M_A(0) \), as a function of \( \bar{\beta}_e \).}
\end{figure}
figures 1(b) and (c), there is a degree of coupling between \( T_\phi \) and \( P_n \), and \( I_p \) and \( B_\phi \). However, the coupling is weak enough to produce a reasonable fit to the data.

The scaling with the density is weakly negative for both the thermal and the Alfvén Mach number. A positive scaling with toroidal torque, \( T_\phi \), and a negative scaling with input power is found. This comes close to the basic trend shown in figure 2(b) for the thermal Mach number. A slightly different dependence with the ratio of torque and input power is found for the Alfvén Mach numbers. Both the thermal and Alfvén Mach numbers scale quite strongly with the inverse safety factor \( q_{ss} \) (i.e. the ratio between \( I_p \) and \( B_\phi \)), especially for the Alfvén Mach number, which scales almost linearly with the inverse \( q_{ss} \). The above shown scaling suggests that there is no plasma rotation (i.e. zero Mach numbers) when no external torque is applied. For some cases, with a limited external torque, plasma rotation was observed as shown in figure 2(b). However, the regression analysis presented here is dedicated to NBI driven rotation and did not attempt to resolve the rotation driven by other means. Details about plasma rotation in JET driven without external torque can be found elsewhere [15,16]. A scaling for the Alfvén Mach numbers in plasmas with no external momentum source was presented in [14]. The scaling shown here, of course, differs due to the strong dependence of the observed rotation on the external NBI torque. A further interesting difference is the opposite dependence on \( q \) between both types of rotation.

The thermal Mach number depends on the plasma rotation as well as ion temperature; a scaling with the total input power is therefore expected. The Alfvén Mach number is based on the plasma rotation only. It is usually observed that the energy confinement scales with the inverse input power. One may argue that a similar trend of the Alfvén Mach number scaling with the inverse input power, suggests that energy and momentum transport are related. Applying more heating power without additional torque, for example by means of ICRH, would reduce the Mach numbers. The lower Mach numbers and angular momentum can be explained by an enhanced turbulence and degradation of confinement by the additional ICRH power flux, similar as discussed in [25].

A similar regression analysis can be carried out to find the scaling of the peaking factor for both the thermal and Alfvén Mach number profiles,

\[
P_{\text{Mth}} \propto n_e^{-0.11 \pm 0.02} \cdot B_\phi^{0.40 \pm 0.05} \cdot T_\phi^{-0.09 \pm 0.04} \cdot P_m^{-0.11 \pm 0.02},
\]

\[
P_{\text{MA}} \propto n_e^{-0.31 \pm 0.02} \cdot B_\phi^{0.37 \pm 0.06} \cdot T_\phi^{-0.08 \pm 0.05} \cdot P_m^{-0.06 \pm 0.03},
\]

where the peaking factors are defined as \( p_{\text{Mth}} = M_{\text{th}}(0)/(M_{\text{th}}) \) and \( p_{\text{MA}} = M_{\text{A}}(0)/(M_{\text{A}}) \). The Pearson correlation coefficients and \( \chi^2 \) for both fits (6) and (7) were found to be \( \rho = 0.57, \chi^2 = 2.44 \) and \( \rho = 0.70, \chi^2 = 1.36 \), respectively.

**Figure 4.** Example of a JET discharge (#51594) that forms an ITB at \( t = 5.86 \) s. (a) Shows a typical optimized shear discharge sequence, with LHCD (green) during the current ramp phase, and a start of the main heating (NBI (blue dotted) and ICRH (red dashed)) just prior to the start of the flat-top. (b) and (c) show the ion temperature and angular rotation frequency, respectively, at various radial positions. (d) The Alfvén Mach number in the core \( (R = 3.1 \text{ m}, \text{red dashed}) \) and at the edge \( (R = 3.77 \text{ m}, \text{black solid}) \). (e) The thermal Mach number in the core \( (R = 3.1 \text{ m}, \text{red dashed}) \) and at the edge \( (R = 3.77 \text{ m}, \text{black solid}) \).

**Figure 5.** Example of a JET discharge (#59646) with a spontaneous transition from type I to III ELMs at \( t = 22.75 \) s, indicated by the dashed vertical line. (a) The top box shows the constant input powers of NBI (blue dotted) and ICRH (red dashed) and the line-integrated density (black solid) (b) The traces of the total angular momentum (red dashed) and the diamagnetic energy (black solid). (c) The average thermal Mach number. (d) The centre \( (R = 3.1 \text{ m}) \) (red dashed) and edge \( (R = 3.77 \text{ m}) \) thermal Mach number. (e) The \( D_\parallel \) trace, indicating the ELM type.
It should be noted that the quality of the fit for the thermal Mach number is rather unsatisfactory. As discussed above an inverse scaling with the density is found. This matches the trends shown in figures 2(c) and 3(c). Both peaking factors depended weakly on the ratio of torque and power.

An attempt was made to study the scaling of the dimensionless Mach numbers with other dimensionless parameters in the database, such as $\rho^*$, $v^*$ and $\beta$, but no satisfactory result was obtained. This may be due to the larger error in these parameters. In addition, the dimensionless parameters are not available for some entries, entailing a degradation of the fit quality due to a low number of used entries. Further work is required to obtain satisfactory scalings with dimensionless parameters.

4. Global confinement of momentum

As seen in the previous section, the magnitude of plasma rotation can be characterized by the Mach numbers. Besides the source (e.g. torque) the rotation is also determined by the plasma viscosity or momentum confinement. The momentum confinement time accounts for this property of the plasma. It is defined as the ratio of the steady-state total angular momentum and the torque applied to the plasma:

$$\tau_p \equiv \frac{\tau_\phi}{\tau_{\phi}} = \frac{L_\phi}{T_\phi} \quad \text{(8)}$$

The confinement time is, in contrast to the Mach number (profile), a global value. It should be noted that the above definition may become invalid for cases with zero external torque input, such as for balanced NBI as shown in [13]. The entries in the JET database always have a finite external torque. It is often observed that the momentum and energy confinement times of tokamak plasmas are similar, indicating that the transport processes for energy and momentum may be linked [3–6]. The steady-state energy confinement time in the rotation database described here is defined as

$$\tau_p \equiv \frac{W_{\text{kin}}}{P_{\text{in}}} \quad \text{(9)}$$

Here, $W_{\text{kin}}$, is the total kinetic energy obtained by integrating the ion and electron pressure profiles. This is a different definition than the one used in the international confinement database, which subtracts the fast-particle energy from the total energy to obtain the thermal energy [18]. It should be pointed out that both methods to calculate the thermal plasma energy do not always provide identical results. For a number of entries from dominant ICRH subset, the duration of NBI, necessary for the CXRS rotation measurement, was shorter or equivalent to the energy confinement time. Hence these data have been omitted from the momentum confinement time analysis.

The momentum and energy confinement times in the database are compared in figure 8(a). It shows that both parameters scale with each other, although large differences exist for individual entries. According to equation (1) it is expected that the thermal Mach number varies with the ratio of energy and momentum confinement time. In figure 8(c) the ratio of both confinement times is plotted versus the average thermal Mach number, showing the variation between the energy and momentum confinement times. The ratio ranges from approximately 0.8–1.4 for most scenarios. Less obvious is the scaling of this ratio with the average Alfvén Mach number, shown in figure 8(d). Note that the data with predominant ICR heating are not following these trends.

Theory on ion temperature gradient (ITG) driven turbulence predicts that the energy and momentum diffusivities are equal [26]. Recent studies at JET have shown that the energy and momentum diffusivity, at least in the plasma core, do not necessary have a one-to-one relationship [10,22]. In figure 8(b), the effective momentum and heat diffusivities differ almost by an order of magnitude. For JET the ratio of the momentum and heat diffusivity, the so-called Prandtl number, is found to be smaller than unity. These effective diffusivities are calculated via the local power and torque balance equations, as discussed in [10]. The local power and torque deposition are determined from the PENCIL code [27]. Both diffusive and convective transport components are combined in these effective diffusivities. It is worth mentioning that due to the inaccuracy of the ion temperature and momentum density gradients, used in the calculation, the resulting diffusivities have considerable errors (>80%). Nevertheless, the fact...
that the overall global energy and momentum confinement times are of the same order of magnitude, while the core diffusivities are not, raises the question of whether there is a distinct difference between core and edge momentum confinement.

Because plasma rotation, or more precisely the rotational shear, is thought to have a stabilizing influence on plasma turbulence [2], the energy and momentum confinement times may depend on plasma rotation. Furthermore, plasma rotation may influence the H-mode pedestal stability and as a consequence the pedestal strength. Usually, these effects may depend on plasma rotation. Furthermore, plasma shear, is thought to have a stabilizing influence on plasma confinement.

4.1. Scaling of confinement times

In the same manner as with the Mach numbers a regression analysis was performed to study the scaling of both the momentum and energy confinement times. Studies into the scaling of the energy confinement time have been performed in great detail [18]. The resulting IPB98c(y,2) scaling was derived from a fit to a large database of ELMy H-mode entries from various devices.

Using the database presented in this paper, the energy and momentum confinement time were first fitted to a model with a non-linear scaling to density, plasma current, magnetic field and input power model, while information on rotation or torque was excluded from this regression analysis. This analysis provided the following results:

\[
\tau_E \propto n_e^{0.41 \pm 0.02} \cdot I_p^{0.76 \pm 0.08} \cdot B_\phi^{0.26 \pm 0.07} \cdot P_{in}^{-0.40 \pm 0.02}.
\]
\[
\tau_p \propto n_e^{0.47 \pm 0.05} \cdot I_p^{1.14 \pm 0.14} \cdot B_\phi^{0.48 \pm 0.14} \cdot P_{in}^{-0.54 \pm 0.05}.
\]

The quality of these fits was reasonable for the energy confinement time with \( \rho = 0.78, \chi^2 = 1.53 \). But the fit for the momentum confinement time proved unsatisfactory with \( \rho = 0.63, \chi^2 = 8.54 \). Clearly the model for the momentum confinement time missed one or more relevant parameters. The result of both fits improved when torque was introduced, yielding, \( \rho = 0.80, \chi^2 = 1.48 \) and \( \rho = 0.74, \chi^2 = 3.96 \) for the scaling of energy and momentum confinement time, respectively. The scaling of both confinement times showed very similar trends with density, plasma current, magnetic field and power. However, an inverse scaling with torque was found for the momentum confinement time while the energy confinement time scaling showed small positive scaling with torque.

The best results were, however, obtained with a model that included the average Alfvén Mach number instead of torque. This gave

\[
\tau_E \propto n_e^{0.37 \pm 0.02} \cdot I_p^{0.56 \pm 0.06} \cdot B_\phi^{0.17 \pm 0.06} \cdot (M_A)^{-0.48 \pm 0.02}.
\]
\[
\tau_p \propto n_e^{0.39 \pm 0.03} \cdot I_p^{0.79 \pm 0.03} \cdot B_\phi^{0.13 \pm 0.11} \cdot (M_A)^{-0.75 \pm 0.04}.
\]

Pearson correlation coefficients and \( \chi^2 \) were found to be \( \rho = 0.86, \chi^2 = 0.99 \) and \( \rho = 0.78, \chi^2 = 3.7 \) for the scaling of energy and momentum confinement time, respectively. The values obtained with the above scaling laws can be compared with the measured values in figure 9. Notable deviations from the momentum confinement time scaling can be seen in figure 9 for discharges with dominant ICRH (overestimation) and those
Figure 8. (a) The momentum confinement time, $\tau_p$, versus the kinetic energy confinement time, $\tau_E$. (b) The effective momentum diffusivity, $\chi_p$, versus the ion heat diffusivity, $\chi_i$, calculated via the local torque and power balance, respectively, for all entries in the database. The local power and torque deposition are determined from the PENCIL code [27]. The data are averaged from $r/a = 0.2$ to $r/a = 0.7$. (c) The ratio of energy and momentum confinement times versus the average thermal Mach number, $\langle M_{th} \rangle$. (d) The ratio of energy and momentum confinement times versus the average Alfvén Mach number, $\langle M_{A} \rangle$. With counter-current NBI (underestimation). This suggests that in these cases either momentum transport differs from the average JET case or that the momentum source is different. For the dominant ICRH discharges one can speculate that this may be due to a possible additional drive, or so-called intrinsic source, such as discussed in [14].

Inserting the scaling for the Alfvén Mach number given by equation (5) into these scalings yields a scaling of confinement times purely based on engineering parameters, including torque. The fact that the momentum confinement time may improve with torque can also be deduced from the trend in figure 1(a) which suggest a non-linear dependence of angular momentum with torque. Furthermore, one can deduce this from the ratio of the scaling laws given by equations (5) and (4). This ratio, which according to equation (3) is proportional to the square root of the toroidal $\beta$ scales with the torque, hence the plasma energy and energy confinement time are also expected to depend on the torque. The model in equations (11a) and (11b) gives a vanishing energy confinement time with vanishing rotation, but one should remember that it is only valid in the operation range of JET with finite values of $\langle M_{A} \rangle$.

It is clear that the scaling for the energy confinement time improved when the Alfvén Mach number was used as a dependent parameter. Both confinement times show a scaling with inverse power which is characteristic for turbulent transport in tokamak plasmas. Except for the addition of Mach number, the basic trends are similar, but not identical, to those in the IPB98(γ,2) scaling [18]. Analysis of the database subset of H-mode only entries proved unsatisfactory, because of the strong coupling between torque and power, as shown in table 3(b). However, the presence of multiple confinement modes in the database is almost certain to negatively affect the quality of the resulting fits. In particular, the use of different confinement modes to break the torque–power correlation is likely to bias the estimated exponents on these two parameters. This reduces the confidence that can be placed in the scalings of equations (10a), (10b), (11a) and (11b). Inclusion of data from H-mode experiments which break the torque–power correlation at JET would resolve this issue.
Hence, the exact details of the scaling coefficients should be taken with care. Furthermore, it does not produce information on the confinement properties for plasmas with no toroidal rotation. Nevertheless, one could conclude that for this database, rotation seems to have an influence on the confinement scaling within the JET operational range that is included in the database. Earlier observations hinted to a difference in energy and momentum confinement. The ratio of the two confinement times shows an inverse trend with the difference in energy and momentum confinement. The ratio of the two confinement times obtained by a regression analysis of the database are shown in equations (11a) and (11b). Because it is rotational shear that may affect turbulence and transport, a more relevant parameter would be the peaking factor of the rotation or Alfvén Mach number profile instead of the average Alfvén Mach number. It was however found that this parameter was not significant to the scaling of the total energy and confinement times. Nevertheless this parameter may have an effect on the core transport as discussed later in this paper.

4.2. Examples

The obtained scaling for the energy and momentum times can be highlighted by a few examples. Looking back to figure 5, in the previous section, a transition from type I to type III ELM My H-mode takes place at $t = 22.75 \, \text{s}$. This transition causes a drop of the thermal Mach number. The transition is also accompanied by a change in the ratio of plasma energy and total angular momentum, while the heating power and external torque remain constant. Clearly the ratio of energy to momentum confinement times increases. This is consistent with the scaling of this ratio with the average Mach number (as shown in figures 8(c) or (d)). The question remains whether the observed change in ratio is due to changes in edge characteristics, reducing the confinement of momentum during the type III phase, or caused by the slower rotation and smaller Mach number.

Figure 9. The values obtained from the scaling laws are plotted versus the measured values; (a) for energy confinement time scaling given by equation (11a) (with a numerical coefficient of 0.43). The Pearson correlation coefficients and $\chi^2$ for this fit were, $\rho = 0.86$, $\chi^2 = 0.99$, (b) for the momentum confinement time scaling given by equation (11b) (with a numerical coefficient of 1.08) with $\rho = 0.78$, $\chi^2 = 3.7$.

Figure 10. Example of a JET discharge (#66016) in which the total heating power is kept almost constant, while the fraction of ICRH and NBI is altered. The fraction of ICRH power to the total changes from 6% to 44%. (a) The top box shows the powers of NBI (blue dotted), ICRH (red dashed) and total power (green dot-dashed) and the line-integrated density (black solid) which remained constant (b) The traces of the total angular momentum (red dashed) and the diamagnetic energy (black solid) (c) The energy (black solid) and momentum (red dashed) confinement times. (d) The centre ($R = 3.1 \, \text{m}$) (red dashed) and edge ($R = 3.77 \, \text{m}$) (black solid) thermal Mach number. (e) The $D_e$ trace, indicating the ELM type.

In figure 10 a discharge is shown where the total heating power is kept constant, while the ratio of NBI and ICRH is altered. The fraction of ICRH power to the total changes from 6% to 44%. The total heating power increases slightly by...
The confinement times discussed above, do not distinguish between core confinement and that provided by an H-mode pedestal. The physics that determine the gradients in both regions, differs, which may be reflected in different scaling for core confinement and that provided by an H-mode pedestal. The confinement times, $	au_E$ and $\tau_P$, are determined for each entry (even those done with co-current NBI). It should be noted that the counter-NBI discharges also exhibit less peaked temperature profiles than those done with co-current NBI.

The torque deposition profile for co-current NBI deviates from counter injection. In the latter case, the NBI generated ions which are trapped in banana-orbits will have an outward radial movement, while for co-NBI this is opposite. Hence, the so-called instantaneous $j \times B$ torque will be more off-axis for counter injection [9]. This is especially evident in counter discharges with high densities, which enhance the off-axis torque deposition. The strongly off-axis torque deposition resulted in a very low momentum confinement times for these discharges. TRANSP analysis showed an off-axis torque profile for this discharge, which was further enhanced during the gas-dosing phase. During the high-density phase shown in figure 11, TRANSP estimated that more than 50% of the total torque is deposited in the outer region of the plasma ($\rho > 0.7$) while for an identical co-injection case this was found to be only 20%. However, the power deposition peaked on-axis, resulting in a peaked temperature and a flat rotation profile.

This example shows that differences in the power and torque deposition can result in different momentum and energy confinement times. The flattening of the rotation and thus Mach profiles at high density is partly revealed by the scaling in the peaking factors (equations (6) and (7)). However, at present torque deposition differences between co and counter injection have not been parametrized in the database.

4.3. Core and pedestal confinement

The confinement times discussed above, do not distinguish between core confinement and core pedestal. The physics that determine the gradients in both regions, differs, which may be reflected in different scaling for the edge and core confinement. The first may be determined by turbulence driven transport while the pedestal gradient could also be limited by MHD stability [28, 29]. This can be reflected in so-called two-term or offset-linear scaling models that treat the scaling of the core confinement independently [18,28]. Similarly as for the plasma energy, this can be used to study the difference between core and pedestal confinement. As has been mentioned above, there are indications that the pedestal confinement for momentum and energy may differ.

The database contains information on the edge or pedestal energy, $W_{\text{ped}}$, and the so-called pedestal momentum, $L_{\text{ped}}^0$. These quantities are determined for each entry (even those without a clear H-mode pedestal) by taking the measurement of the kinetic pressure and momentum density at the edge of the plasma ($r/a = 0.89$) multiplied with the plasma volume (because the volume contained by this region is in fact slightly smaller it is actually multiplied with 95% of the volume). The fixed position is chosen in order to simplify the calculation of these parameters for all database entries. It coincides with a major radius of approximately $R \sim 3.8 \text{ m}$, which is close to the location of the outer most reliable ion temperature measurement. The width of the H-mode pedestal is not known for most of the database entries. Another issue

![Figure 11](image-url)
Figure 12. (a) The pedestal (edge) momentum versus the pedestal energy. The scaling given by equation (12) is represented by the solid red line. (b) Comparison of the pedestal (edge) momentum and energy confinement times. (c) Comparison of the core momentum and energy confinement times. (d) The ratio of the core energy and momentum confinement times as a function of the average thermal Mach number.

is the time resolution of the CXRS diagnostic that measures the rotation and ion temperature. The edge parameters $L_{\text{ped}}$ and $W_{\text{ped}}$ are averaged over a period that could experience several ELM pedestal collapses. All these uncertainties make that the percentage error for the pedestal parameters is higher than those of the total integrated energy and angular momentum (see table 2).

In figure 12(a) the pedestal momentum is found to follow pedestal energy. This is predominantly a consequence of the link between heating and torque sources in the JET data. The highest pedestal values are of course found for the H-mode entries. A simple fit to the data gives

$$L_{\text{ped}} = 0.69 \pm 0.08 \cdot W_{\text{ped}}^{1.1(1)}.$$  \hspace{1cm} (12)

With a Pearson correlation coefficient of $\rho = 0.82$, with $L_{\text{ped}}$ in (kg m² s⁻²) and $W_{\text{ped}}$ in (MJ). The fraction of energy in the pedestal almost always exceeds that of the fraction of momentum in the pedestal. This is especially true for the type I ELMy H-mode discharges. For these H-mode entries the average pedestal energy fraction is of the order of 60% while the fraction of the pedestal to the total momentum confinement is approximately 40%. Notably quite a number of counter discharges do not follow this trend.

The energy and momentum stored in the core, $W_{\text{core}}$ and $L_{\phi}$, respectively, are found by subtracting the pedestal values from the total, $W_{\text{kin}}$ and $L_{\phi}$. The two-term energy and momentum confinement time separating the core and pedestal physics can be defined as

$$\tau_E = \frac{W_{\text{kin}}}{P_{\text{in}}} = \frac{W_{\text{core}} + W_{\text{ped}}}{P_{\text{in}}} = \tau_{E_{\text{core}}} + \tau_{E_{\text{ped}}},$$  \hspace{1cm} (13)

$$\tau_{\phi} = \frac{L_{\phi}}{T_{\phi}} = \frac{L_{\phi_{\text{core}}} + L_{\phi_{\text{ped}}}}{T_{\phi}} = \tau_{\phi_{\text{core}}} + \tau_{\phi_{\text{ped}}},$$  \hspace{1cm} (14)

The comparison of these parameters provides interesting information about the differences between the confinement of energy and momentum in the core and edge or pedestal region. For most of the predominantly NBI heated H-mode entries the ratio of torque and input power is of the order of $T_{\phi}/P_{\text{in}} \approx 1.1 \pm 0.15$ (N m MW⁻¹). With this information equation (12) therefore indicates that the pedestal confinement of momentum is lower than that for the energy. It should be
noted that a split between the core and pedestal momentum confinement times is only possible when the plasma rotation is uni-directional, which is the case for standard JET operations (see figure 12(a)).

In figures 12(b) and (c) the momentum and energy confinement times for the edge (pedestal) and core are compared, respectively. Figure 12(c) shows that the edge momentum confinement time is significantly smaller than that of the energy. The H-mode pedestal mainly improves the energy confinement. The reverse is found in the core, quite a large number of entries, mainly H-modes, show a larger confinement for momentum than energy in the core. This is consistent with earlier observations that in JET the effective momentum diffusivity in the core is smaller than that for the ion heat diffusivity [10]. The small Prandtl numbers found in JET are due to a combination of a difference in core momentum and energy confinement times, as well as larger gradient lengths for the ion temperature profile compared with that of the momentum density. The total confinement times for momentum and energy may still have similar magnitudes, such as seen in figure 8(a), as the differences in core and edge confinement seem to compensate each other.

In the example in figure 11 it turns out that it is the core momentum confinement time that is degraded while the edge/pedestal confinement stays largely unchanged by the increased density. In figures 8(c) and (d) it was shown that the momentum and energy confinement time differ due to variations in the rotation itself. It turns out that this effect is due to core physics as it is the ratio of core momentum and energy confinement times that is changed by Mach number, as shown in figure 12(d), while the edge confinement times were found to be unaffected. Figure 12(d) shows that the ratio of core energy and momentum confinement time is often smaller than unity (i.e. momentum confinement is better) and that this ratio decreases for larger thermal Mach numbers. It was recently shown that the presence of a momentum pinch could reduce the effective momentum diffusivity in the plasma core, hence increasing the core momentum confinement time with respect to that of the energy [30]. The observed trend in figure 12(d) is in agreement with the studies shown in [22, 30, 31], which predict a smaller effective momentum diffusivity for larger Mach numbers due to the presence of an inward pinch.

Changes in the pedestal confinement of momentum are also responsible for the differences observed between type I ELMy H-mode discharges and those with type III or compound ELMs as shown in figures 5 and 6. The transition from type I to type III ELMs shown in figure 5, causes a 40% reduction in the pedestal momentum confinement (from $\tau_{\text{ped}} = 0.083$ s to $\tau_{\text{ped}} = 0.049$ s) while the core momentum confinement time remains unchanged at $\tau_{\text{core}} = 0.036$ s. The transition causes only a 12% reduction in energy confinement ($\tau_{\text{ped}}^E = 0.18$ s to $\tau_{\text{ped}}^E = 0.16$ s). Similarly, the compound phases in figure 6 are characterized by a low pedestal momentum confinement time, while the core momentum confinement is more or less unchanged over these transitions.

5. Discussion

A large steady-state rotation database including all plasma scenarios, a compromise between parameter accuracy and large number of entries, has been built at JET. This database proved efficient in identifying broad trends in plasma behaviour as concerns rotation, although its quality is still affected by problems inherent to large databases, such as correlations or clusters in parameter space. This paper reported on observed trends found with this database and gave an overview of the general characteristics of toroidal plasma rotation in JET.

An offset scaling of central velocity with central ion temperature was found, with a similar trend for the global values of total angular momentum and kinetic energy. Profile average thermal Mach numbers of about $M_{th} = 0.33$ are observed on JET, roughly scaling with total power divided by torque. The Alfvén Mach number is one order of magnitude lower and is observed to scale with $\beta_{\phi}$. Type I ELMy H-modes have the highest values, whereas both Mach numbers in type III ELMy H-modes are significantly lower. $M_{th}$ values are higher in ITB discharges than other scenarios, while this is not observed for $M_{th}$. Both Mach numbers profiles are less peaked in high-density discharges and can become hollow for predominantly ICRH-heated and counter-NBI shots. For those discharges with minimum external torque ($T_{\phi} < 1$ N m) the Alfvén Mach numbers were in the range $0.0009 < M_{th} < 0.008$. The question is how these values relate to the intrinsic rotation that is observed in other devices [14].

Regression analyses were carried out on both Mach numbers. The best fit to the data was found using tokamak engineering parameters. They showed a negative scaling with line-averaged density, toroidal magnetic field and total power and a positive scaling with plasma current and torque, as shown by equations (4) and (5). It is interesting to note that the average Alfvén Mach number scales almost linearly with the NBI torque. If one assumes that this torque is the only source of the rotation, the other parameter scalings reflect transport effects. This is already indicative from the $P^{-0.36}$ scaling similar to that found in for example the scaling of energy confinement (equation (10e)). The Mach profile shape depends strongly on the density, as more torque is deposited off-axis for high-density JET discharges.

As concerns transport, the energy and momentum confinement times were found to be approximately equal. The $\tau_{\phi}/\tau_{E}$ ratio scales with the thermal Mach number, as expected by definitions of these parameters. This ratio also scales with the Alfvén Mach number, although this parameter does not include energy or power, hinting to a role of rotation in confinement properties. When a rotation scaling parameter was introduced, regression analysis found an improved scaling for both the energy and momentum confinement times (see equation (11a) and (11b)). Both scale positively with the average Alfvén Mach numbers. Although the coefficients for both scaling laws are similar, the momentum confinement time seems to have a stronger negative dependence on power, and a stronger positive scaling with plasma current and Mach number.

Coupling exists between various parameters, such as torque and power, which complicated the regression analysis. A principal component analysis, similar to that discussed in [18], was carried out, indicating that the database had only three well conditioned components. If the subset of H-mode only entries are considered the coupling between
torque and power further reduces the number of principal components to two. This shows that the database presented in this paper is less well defined than the energy confinement database used to determine the H98(y,2) scaling which has five well-conditioned principal components [18]. Therefore, the predictive power of these scaling laws is limited although they show trends within the JET parameter range. A better understanding of these scaling of rotation and momentum confinement, especially for the H-mode subset, would require further experiments designed so as to break the correlation between power and torque, for example by increasing the fraction of ICRH power in JET plasmas.

Distinction of the core and pedestal contributions to the energy and momentum revealed the difference in confinement of these regions. An H-mode pedestal leads to a larger increase in $W_{\text{ped}}$ than $L_{\phi,\text{ped}}$, whereas the core values remain unaffected. This is consistent with the difference in average Mach numbers in type I and III ELMy H-modes, which is predominantly caused by edge differences. The edge momentum confinement time improves less than the edge energy confinement time in the presence of an H-mode pedestal. The pedestal energy and momentum confinement are clearly governed by different physics, where the pedestal viscosity may possibly be influenced by edge radial electric fields, error fields, interaction with neutrals, etc, effects that do not play a role in the energy pedestal.

The ratio of the core energy and core momentum confinement times scales with the Mach number. It has been shown that the effective momentum diffusivity is lower than the heat diffusivity in the core which leads to a larger core momentum confinement time compared with that of the energy. It was recently shown that an inward momentum pinch could explain the improved core momentum confinement [30, 31]. Experimental indications of such a momentum pinch were found at JET [22]. The better momentum confinement in the core balances the lower edge momentum confinement, which explains why the ratio of total energy and momentum confinement time has approximately the same magnitude. Studies into the difference between core and pedestal momentum confinement would benefit from experiments in which these parameters can be determined with a better accuracy.

A number of counter-current NBI discharges were found to deviate from other entries. It was already pointed out that these discharges often exhibit hollow Mach profiles. These emphasized the importance of profile effects. Generally, discharges with counter-current NBI exhibited a smaller angular momentum than those with similar toroidal torque but co-NBI. However, the database was not equipped with sufficient information to distinguish trends related to differences in torque and power deposition. This urges to include more detailed profile information for further transport studies.

The JET rotation database enables the identification of the role played by rotation in confinement. Nonetheless, these results are to be considered carefully. Rotation can differ from device to device due to differences in heating systems, error fields, toroidal field ripple or edge viscosity driven by neutrals. It has been shown that TF ripple can have a profound influence on plasma rotation [24, 32]. Of particular importance is the orientation of NBI, ranging from normal to tangential, and their direction relative to the plasma current, counter or co-injection. Recent experiments at DIII-D showed that with balanced NBI operation and zero integrated torque input, the plasma may still have a significant rotation [13]. The presence of an intrinsic momentum source is important for momentum transport studies where a proper understanding of the source is significant. This database is limited to a single machine, meaning a restricted region in parameter space; hence the scaling laws derived by regression are valid only in the vicinity of this region.

With a larger moment of inertia but a torque only twice as high than JET, it is expected that ITER will rotate slower, i.e. smaller average Mach numbers. The higher fraction of ICRH may result in hollow Mach profiles and off-axis torque deposition may enhance this effect. The Toroidal Field ripple will also be considerably higher in ITER compared with JET [31]. All these factors make it difficult to extrapolate the experimental results presented in this paper to ITER. An extension of the database with data from other devices would be beneficial to enable confident ITER extrapolations.

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Saturated internal instabilities in advanced tokamak plasmas

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‘Advanced tokamak’ (AT) scenarios were developed with the aim of reaching steady-state operation in future potential tokamak fusion power plants. AT scenarios exhibit non-monotonic to flat safety factor profiles (q, a measure of the magnetic field line pitch), with the minimum q (q_{min}) slightly above an integer value (q_s). However, it has been predicted that these q profiles are unstable to ideal magnetohydrodynamic instabilities as q_{min} approaches q_s. These ideal instabilities, observed and diagnosed as such for the first time in MAST plasmas with AT-like q profiles, have far-reaching consequences like confinement degradation, flattening of the toroidal core rotation or enhanced fast ion losses. These observations motivate the stability analysis of advanced tokamak plasmas, with a view to provide guidance for stability thresholds in AT scenarios. Additionally, the measured rotation damping is compared to the self-consistently calculated predictions from Neoclassical Toroidal Viscosity theory.

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A potential tokamak power plant would greatly benefit from steady-state operation and a high ratio of plasma stored energy to magnetic energy (defined as $\beta = 2\mu_0 p / B_x^2$, with (p) the volume average pressure and $B_x$ the central toroidal magnetic field). Since the limited duration of baseline tokamak plasmas arises from the inductive drive of the toroidal plasma current, the objective of ‘advanced tokamak’ (AT) scenarios [1, 2] is twofold: sustaining high plasma $\beta$ and optimising the non-inductive, self-generated ‘bootstrap’ current. This current is predominantly driven off-axis, hence significantly altering the magnetic field pitch angle, expressed in terms of the safety factor (q, the inverse average pitch angle). In the ‘steady-state’ scenario [1], it provides more than half of the total current, producing a reversed shear (non-monotonic) q profile, the magnetic shear being defined as $q^{-1} dq/dr$ with r the minor radius. While not envisaged as steady-state for ITER, the ‘hybrid’ scenario has a significant bootstrap current fraction, hence an increased discharge length and a weakly reversed to broad low shear q profile [2]. These profiles, together with the high $\beta$ characteristic of AT scenarios, render the plasma prone to the deleterious resistive wall mode instability [3], which was fortunately shown to be stabilised by various effects [4–6]. Due to the low core current, the minimum value of the q profile (q_{min}) is above an integer value q_s, meaning $\Delta q = q_{min} - q_s > 0$. This, for example, prevents the occurrence of deleterious resistive MHD instabilities such as sawteeth at q = 1, or neoclassical tearing modes (NTM) at low order rational q surfaces [2]. AT plasmas have however been predicted potentially unstable to ideal MHD modes when $\Delta q$ approaches zero [7, 8]. For the first time, such instabilities have been observed as long-lived saturated modes in MAST plasmas with reversed shear q profiles and positive $\Delta q$ close to zero. These modes significantly deteriorate the confinement, damp the core rotation and enhance fast ion losses. This letter details the analysis of these saturated instabilities and their consequences from a theoretical, modelling and experimental point of view. Such consistent understanding provides guidance for stability thresholds of AT scenarios and may shed light on saturated long-lasting modes in other tokamaks.

In MAST, AT-like q profiles are obtained in low density discharges heated by neutral beam injection (NBI). High core temperatures result in a decreased resistivity, delaying the penetration of the inductively-driven current into the core and thus creating a weakly reversed q profile (figure 1). The latter is obtained from an EFIT equilibrium reconstruction [9], constrained to pressure profiles deduced from Thomson Scattering (TS), Charge eXchange Recombination Spectroscopy (CXRS) and $Z_{eff}$ measurements, and the magnetic pitch angle profile from the newly installed Motional Stark Effect diagnostic (MSE), with a resolution ≤ 2.5 cm (≤ 5% of the minor radius). In these plasmas, $\Delta q$ decreases slowly to become close to zero for most of the discharge. Despite the uncertainty inherent in the EFIT reconstruction, there is great confidence that $\Delta q$ stays positive, i.e the q profile is above 1, since otherwise the sawtooth instability is predicted to be unstable. The early phase of the shot often exhibits frequency chirping fast ion driven instabilities, also known as ‘fishbones’, of toroidal periodicity n = 1. For high $\beta$ plasmas, $\Delta q$ evolves towards zero, a saturated long-lived mode (LLM) is observed on the spectrogram of outboard mid-plane Mirnov coils (magnetic probes) measurements, and persists until the termination of the plasma, i.e for several confinement times or $\sim 10^6$ Alfvén times (figure 2a). Simultaneously, a degradation of energy confinement occurs (figure 2b), together with a flattening of the rotation profile measured by CXRS (figure 2c), and enhanced fast-ion losses, as indicated by counter-
viewing bolometer data. While experimentally varying the LLM onset time by changing the NBI heating waveform or the toroidal field, hence the profile, these deleterious effects always occurred simultaneously to mode onset, providing strong evidence that they result directly from the LLM. Here, the energy confinement time is calculated using the TRANSP code [10], accounting for the LLM-induced fast ion losses with an anomalous fast ion diffusion, the magnitude of which is chosen to match the measured and predicted neutron production rate.

Although saturated modes are most often resistive (ie associated with tearing of magnetic surfaces and reconnection to form magnetic islands [11]), experimental data give no evidence of magnetic reconnection. No local flattening is detected on high resolution TS profiles. Additionally, the induced perturbation in the Soft X-Ray (SXR) emissivity, which is predominantly dependent on electron temperature, would change sign across a magnetic island. In the presence of an island and without diffusion, the magnitude of which is chosen to match the measured and predicted neutron production rate.

AT-like profiles are predicted to be ideally MHD-unstable. Theory analysing the reversed shear profile indicates that it is prone to the \((m, n) = (1, 1)\) internal kink mode [7]. The mode is unstable, even at zero \(\beta\), for \(\Delta q\) under a critical value \(\Delta q_{\text{crit}}\) calculated analytically for low inverse aspect ratio plasmas in [7]. This mode saturates non-linearly if the \(q\) profile remains above 1 [12]. The mode being ideal, the saturation occurs when the stabilising field line bending term balances the mode’s fluid drive. Theory focusing on flatter core \(q\) profiles predicts such plasmas to be unstable to low \((m, n)\) internal kink-balloonning instabilities, called internal modes [8, 13], the most unstable modes having \(m = n\) when \(q_{\text{min}} \sim 1\). These modes also feature a critical value of \(\Delta q\) under which they are destabilised [8]. This critical value decreases with the \(n\) number, whereas the mode’s growth rate increases with it at \(\Delta q \approx 0\). Internal modes in AT plasmas have been the subject of much theoretical research [13, 14] and found to disrupt plasmas in several tokamaks [15–17].

Both the internal kink and internal modes are very similar in their drive, structure and threshold, differing only in the magnetic shear assumed in their respective theories. Since the \(q\) profile in LLM plasmas evolves from reversed to flat shear, it seems unnecessary to distinguish between these modes in the following discussion. These modes are more unstable in tight aspect ratio geometries, making MAST ideally-suited to study them. Their predicted triggering below a certain \(\Delta q\) is in excellent agreement with experimental LLM onset time, and the SXR fluctuation simulated from their theoretical structure is consistent with those observed (figure 3). Assuming that the \(n = 1\) and \(n = 2\) components of the experimentally observed LLM both resonate at the \(q_{\text{min}}\) surface (as predicted by the MISHKA-1 MHD code [18]), the relative

\[ \psi N \in \frac{1}{2} \, 0 \, 1 \, 2 \, 3 \, 4 \, 5 \]

\[ \text{LLM shot #21781} \]
amplitude of these components can be inferred from the peaks at the fundamental and first harmonic frequencies in the SXR spectrogram. These show that the \( n = 2 \) component is not detected at LLM onset, whereas it appears and grows in amplitude as \( \Delta q \) decreases (figure 2d). Whilst the \( n > 1 \) modes initially arise as a nonlinear consequence of the \( n = 1 \) mode, the change in their relative amplitude is likely to occur because of a resonant field amplification effect \([19]\) in the higher \( n \) harmonics as they become marginally unstable. Once their respective \( \Delta q_{\text{crit}} \) is reached, the \( n > 1 \) modes are destabilised and grow. Since theory suggests that the \( n = 1 \) mode is unstable at larger \( \Delta q \) than instabilities with \( n > 1 \), but that the latter dominates as \( \Delta q \) approaches zero, this gives increased confidence in the interpretation of the LLM as an ideal mode.

The linear stability of MAST plasmas with AT-like \( q \) profiles was analysed with the MISHKA-1 code for different toroidal mode numbers. For each \( n \), the most unstable mode predicted by MISHKA-1 has a kink-ballooning mode structure with dominant \( m = n \), a structure similar to that observed on SXR data and those predicted by theory. In order to investigate the stability of equilibrium with different \( \Delta q \), the toroidal field given as input to MISHKA-1 was scaled accordingly. Figure 4 shows the growth rate of the \( n = 1, 2, 3 \) modes as a function of \( \Delta q \). It is evident in this figure that \( \Delta q_{\text{crit}} \), the critical stability threshold, decreases with \( n \), whereas the mode’s growth rate increases with it at \( \Delta q \sim 0 \), consistent with the theoretical features of the ideal internal modes presented above, and experimental observations (figure 2d). Furthermore, analysis also indicates that at low \( \beta \), AT equilibria are linearly stabilised by toroidal plasma rotation. Since rotation in MAST can be a significant fraction of the ion sound speed, this helps to explain why low \( \beta \) AT-like plasmas do not exhibit the LLM. Simulations performed with the HAGIS drift-kinetic code \([20]\) showed that fast ions stabilise the LLM, with a reduction of \( \Delta q_{\text{crit}} \) of the order of 10%. Calculation using the MISHKA-D code \([21]\) identified ion diamagnetic effects as another second order stabilising mechanism.

AT scenarios have also been found to be more unstable to fast-ion driven fishbone instabilities, with serious consequences for fusion performance in future fusion devices, as these instabilities could expel energetic \( \alpha \) particles before they heat the plasma. AT-like MAST plasmas often feature bursts of fishbone instabilities, which cease at the onset of the saturated mode. For reversed shear plasmas with \( \Delta q > 0 \), the reduced Alfvén continuum damping at the \( q_{\text{min}} \) surface makes fishbones unstable at lower fast ion pressures. This increased susceptibility to fast ion-driven fishbones could be important in ITER where fusion-born \( \alpha \) particles will provide a strong drive for the mode. In addition, calculations predict that fishbones are unstable at much higher \( \Delta q \) than the ideal mode. This is in good agreement with the experimental observation of an early fishbone phase followed by LLM onset, after which the fishbone activity stops due to the suppression of its drive by enhanced fast ion losses, as observed on bolometer data.

Following the onset of the LLM, the rotation frequency of the mode remains constant (figure 2a) while that of the plasma core is rapidly damped (figure 2c). Electromagnetic torques can appear even in the absence of magnetic reconnection \([22]\). They are nonetheless localised around integer \( q \) positions, which, in discharges featuring the LLM, are located outside the region of rotation damping, and beyond mid-radius. While these torques may account for the slight edge rotation acceleration, they cannot explain the core damping, as MAST momentum confinement times (\( \sim 50 \text{ms} \)) do not allow such localised slowing down to diffuse into the core on the observed
timescales ($\sim 5$ms). In contrast, the distributed, fast damping mechanism by Neoclassical Toroidal Viscosity (NTV) theory [23], arising from the non-axisymmetry of the magnetic field, seems best suited to account for the observed braking. The saturated amplitude is estimated by comparing experimental SXR data to simulations for different eigenstructure amplitudes (figure 3), enabling the calculation of the braking torque according to the NTV theory. Except at the inertial and rational surfaces, the results show strong similarities with the measured rate of change of angular momentum density (figure 5). The latter is defined by

$$I_\phi \approx m_i n R^2 \omega,$$

where $m_i$, $n$, $R$ and $\omega$ are the ion mass, density from TS, flux surface average major radius from EFIT and the angular frequency from CXRS. Here, the changes to the mode's eigenstructure as it saturates, which are likely to be small [12, 24], and the effect of enhanced fast ion losses are neglected. While previous work used NTV theory in plasmas with static external magnetic perturbations [25], calculating rotation damping due to saturated internal modes is a somewhat novel application for this theory, in which the response of the plasma to the magnetic perturbation is determined self-consistently. The equivalence to the static case is found by going to the frame co-rotating with the mode, the angular frequency of which is determined using the SXR data.

This letter reports the first analysed observation of a saturated long-lived mode (LLM) occurring in plasmas with AT $q$ profiles, causing significant confinement degradation, core rotation damping and enhanced fast ion losses. While such a saturated mode is most unstable in tight aspect ratio geometries, similar MHD instabilities were observed in JET [26], FTU [27], NSTX [28, 29], JT-6U [30] and DIII-D [31–33]. In ITER, the hybrid scenario is designed to have a broad low shear region with $q_{\text{min}} \gtrsim 1$; in the light of this study, this scenario may also have to operate at a $\Delta q$ value large enough to avoid deleterious saturated modes like the LLM.

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Comparison of MHD-induced rotation damping with NTV predictions on MAST

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Abstract
Plasma rotation in tokamaks is of special interest for its potential stabilizing effect on micro- and macro-instabilities, leading to increased confinement. In MAST, the torque from neutral beam injection can spin the plasma to a core velocity $\sim 300$ km s$^{-1}$ (Alfvén Mach number $\sim 0.3$). Low density plasmas often exhibit a weakly non-monotonic safety factor profile just above unity. Theory predicts that such equilibria are prone to magneto-hydro-dynamic (MHD) instabilities, which was confirmed by recent observations. The appearance of the mode is accompanied by strong damping of core rotation on a timescale much faster than the momentum confinement time.

The mode's saturated structure is estimated using the CASTOR code together with soft x-ray measurements, enabling the calculation of the plasma braking by the MHD mode according to neoclassical toroidal viscosity (NTV) theory. The latter exhibits strong similarities with the torque measured experimentally.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Plasma rotation has recently received growing interest as it proved beneficial in achieving better confinement in tokamaks [1]. On a micro-scale, the resulting $E \times B$ flow shear helps suppress the turbulence [2, 3], while on the macro-scale, rotation exerts a stabilizing effect on magneto-hydro-dynamic (MHD) modes. This is observed for a large range of instabilities, including the ballooning mode [4–8], the internal kink mode [9, 10] and the sawtooth instability [11–13].
Advanced scenarios for plasma operation being presently developed for ITER [14–18] aim at high fusion gains and steady-state operation. They rely on an optimized self-generated plasma current, the so-called bootstrap-current [19], to produce steady-state discharges. In the hybrid scenario [20, 21], a broad low shear or weakly reversed shear safety factor \( q \) profile is used towards this goal, the magnetic shear being defined here as \( s = rq^{-1} dq/dr \). Such safety factor profiles also have a minimum value just above 1, which prevents the appearance of the sawtooth instability, detrimental for plasma confinement. In high performance plasmas, however, this scenario suffers from the resistive wall mode (RWM) arising from the interaction of a pressure-driven external kink instability with the tokamak wall [22]. A relevant performance measure is the plasma \( \beta \) defined as \( 2\mu_0 \langle p \rangle /B_0^2 \), angled brackets denoting a flux surface average and \( B_0 \) the toroidal magnetic field at the magnetic axis. Sustaining high toroidal rotation is of importance when dealing with the RWM, since it is one of the ways of operating above the mode’s \( \beta \) threshold [23–29], and reaching the targeted high pressures.

Over the past few years, a number of deleterious phenomena have been observed, whereby electromagnetic perturbations degrade plasma rotation, the latter change further amplifying the original instability, leading to a self-feeding process which destroys plasma confinement. This is observed at the edge of the plasma, where penetration of error-fields can occur [30], or in the triggering process for edge localized modes [31]. It can also affect the whole plasma, for example in the form of mode locking [32], the drag being associated with mode-induced eddy currents penetrating into the tokamak’s coils and vessel. Such effects with far-reaching consequences on plasma performance justify the investigation of the interplay between MHD activity and plasma rotation.

Neutral beam injection (NBI) in MAST provides ideal conditions for such studies. The torque it provides spins the plasma core up to velocities \( \sim 300 \text{ km s}^{-1} \) which corresponds to a core Alfvén Mach number \( \sim 0.3 \). The latter is defined as the ratio of the plasma velocity to the Alfvén velocity, \( M_A = v_\phi B_0^{-1} (\mu_0 \rho)^{1/2} \), with \( v_\phi \) the toroidal plasma velocity and \( \rho \) its mass density. The increased core temperature resulting from the NBI heating slows down current diffusion, producing a hybrid-like, broad low shear or weakly reversed shear \( q \) profile just above 1. Equilibria with such \( q \) profiles are predicted to be prone to saturated ideal-MHD instabilities when the minimal \( q \) value, \( q_{\text{min}} \), approaches an integer value [33–36]. This type of instability is indeed observed in MAST when \( q_{\text{min}} \sim 1 \) [37] and is referred to as long-lived mode (LLM).

Following the onset of the LLM, strong damping of core rotation is observed on a timescale much shorter than that of momentum transport (figure 1). During this phase, the mode angular frequency does not evolve significantly. After the plasma angular frequency profile has become flat, both the rotation of the plasma and that of the mode gradually decrease.

This study focuses on the initial flattening of the rotation profile. Its short timescale compared with that of momentum transport implies that the braking of the plasma does not arise from MHD-enhanced momentum transport. The LLM causes fast ion losses and redistribution, which affects rotation. Initial calculations indicate that the torque associated with this effect is significantly smaller than that needed to produce the observed braking. Further calculations using the HAGIS drift-kinetic code [38] will be performed in the future to assess the effect of the fast ion redistribution on rotation. High resolution temperature measurements using Thomson scattering (TS) do not show any local profile flattening characteristic of magnetic islands. Additionally, no \( \pi \) phase jumps are observed between neighbouring channels of poloidal cross-section soft x-ray cameras (SXR). These phase jumps are another indication of magnetic reconnection, and are indeed observed in MAST in the presence of neoclassical tearing modes (NTMs). This does not only corroborate the ideal-MHD nature of the mode, but also rules out resonant electromagnetic torques associated with reconnection as a mechanism for plasma...
braking [39, 40]. Electromagnetic torques can act on the plasma even in the absence of magnetic islands [41]. They are nevertheless strongly localized around the Alfvén resonances, in the vicinity of the minimal and integer $q$ locations, inconsistent with the collapse of the whole rotation profile caused by the LLM. In plasmas featuring the LLM, the innermost Alfvén resonance is located around mid-radius, meaning that electromagnetic torques would result in a maximal damping outside the core of the plasma, in contradiction with the observations. Significantly affecting core rotation with these torques would require a diffusion about an order of magnitude higher than observed before mode onset, and the resulting damping profile would still not be peaked in the core, as seen in the experiment. In contrast, the torque arising from neoclassical toroidal viscosity (NTV) [42] is distributed and occurs on a thermal ion collision time scale ($\sim 10^{-4}$ s), thus appearing well suited to describe the measured braking of the plasma by the LLM.

The research presented here investigates whether the torque predicted by NTV theory is consistent with the observed damping of core rotation following the LLM onset. The structure of the LLM is first determined using the CASTOR linear code [43], its saturated amplitude is then estimated by comparing results of forward simulations of the SXR emission with the measurements. This is detailed in section 2. Based on this information, the NTV torque is calculated and compared with experimental data, a process which is explained in section 3. Figure 2 summarizes the rationale of the analysis described through sections 2 and 3. Section 4 presents results of experiments carried out on MAST, while section 5 draws conclusions.
2. Structure of the MHD mode

The structure of the LLM is investigated using the CASTOR code (section 2.1). It is a linear code, therefore giving the eigenstructure of the most unstable mode, but without any information on the saturation amplitude. MHD modes can be observed on the SXR diagnostic (section 2.2), which can yield an estimate of the saturated amplitude. This is done, given the eigenstructure calculated by CASTOR, by simulating the observed SXR fluctuations for a range of assumed amplitudes and selecting the best agreement with the experimental data (sections 2.3 and 2.4). The upper part of figure 2 summarizes this process.

2.1. Eigenstructure analysis from the CASTOR code

The equilibria analysed here are reconstructed using the EFIT code [44], constrained to magnetic field pitch-angle data from the motional Stark effect diagnostic (MSE), as well as total pressure data measured by TS, charge exchange recombination spectroscopy (CXRS) and bremsstrahlung measurements. This enables an accurate calculation of the q profile of the discharge (figure 3). The CASTOR code requires the equilibrium input to be calculated by the HELENA code [45], a fixed boundary Grad–Shafranov equation solver employing the same straight field line coordinates as in the linear stability analysis. The HELENA equilibrium is adjusted to match the q profile reconstructed by EFIT.

Theory predicts the broad low shear or reversed shear equilibria studied to be unstable to ideal modes [33]. In addition, experimental observations do not give any evidence of the presence of reconnection, neither in the form of local flattening of high resolution TS electron temperature profiles, nor in the form of π phase jumps in neighbouring channels of poloidal cross-section SXR cameras (section 2.2). The resistivity in CASTOR is therefore set to zero. Although the MHD mode analysed here was shown to interact with fast ions [37], this effect was not modelled while using CASTOR. The stability analysis is done in full toroidal geometry, taking into account plasma shaping. For a given toroidal n number, it assumes a spectrum of poloidal m numbers with \( m \leq 30 \), a spectrum broad enough to represent core instabilities. The equilibrium at mode onset reconstructed by the HELENA code is found to be unstable to an \( n = 1 \) ideal internal mode, consistent with [33].

2.2. Observation of MHD on SXR

In the case of MAST’s horizontal SXR cameras array, the lines of sight along which the light is collected are located on a poloidal cross-section, at a fixed toroidal position (figures 4 and 5).
In the frequency range of the cameras, the SXR emissivity of the plasma is given by [47]

\[ \epsilon_{\text{SXR}} \propto \frac{n_e n_i Z_{\text{eff}}^2}{\sqrt{T_e}} \int e^{-h\nu/eT_e} \, d\nu, \]  

where \( n_e, n_i, Z_{\text{eff}}, e, T_e \) and \( \nu \) are, respectively, the electron density, ion density, effective ion charge, the elementary charge, electron temperature in eV and frequency of emission.
Equation (1) shows that the SXR emissivity is predominantly dependent on $T_e$ and increases with it. In the case of a non-axisymmetric plasma, as the magnetic structure advected by the plasma motion flows past the cameras (figure 5), the temperature perturbation associated with the magnetic structure results in the fluctuation of the SXR signal in time, allowing the observation of MHD on SXR measurements. SXR data exhibit an additional feature when a magnetic island is present in the plasma: a characteristic $\pi$ phase jump is observed between the two neighbouring lines of sight viewing opposite sides of the island. This is because the temperature perturbation changes sign across the island, and consequently the two lines of sight measure fluctuations of opposite sign [46]. Such $\pi$ phase jumps are observed in MAST in the presence of NTMs, but not in plasmas with the LLM. Note, however, that if an island were present and led to local impurity accumulation, the resulting parasitic line emission would screen the phase jump and would thus prevent the detection of the island.

2.3. Simulation of SXR measurements

Equation (1) is used to express the SXR emissivity as a function of poloidal flux, $\varepsilon_{SXR} = f(\psi)$. Mid-plane TS measurements yield $T_e(\psi)$, $n_e(\psi)$ and $n_i(\psi)$ (assuming quasi-neutrality) and $Z_{\text{eff}}(\psi)$ is given by the analysis of bremsstrahlung emission on an equilibrium timescale.

The eigenstructure of the $n = 1$ ideal mode is given by CASTOR. Assuming an arbitrary amplitude makes it possible to calculate the poloidal flux perturbation for the entire three-dimensional plasma: $\delta\psi = f(R, \phi, Z)$ (with $(R, \phi, Z)$ the usual cylindrical coordinates). Added to the equilibrium flux, this provides the complete topology of the plasma flux surfaces, $\psi = f(R, \phi, Z)$, thus yielding a three-dimensional map of the SXR emissivity of the plasma: $\varepsilon_{SXR} = f(R, \phi, Z)$.

The reconstruction described above is valid for a non-rotating magnetic structure. In reality, the structure moves toroidally due to plasma rotation, and its motion with respect to the plasma. In order for the MHD mode not to lose coherence, this motion has to be rigid rotation, such that $\psi = f(R, \phi + \omega t, Z)$, where $\omega$ is the angular frequency deduced from the experimental data by Fourier transform and $t$ the time. The magnetic structure therefore moves across the camera plane located at $\phi = \phi_0$ and the SXR emission on that plane is
\[ \epsilon_{\text{SXR}} = f(R, \varphi_0 + \omega t, Z) \]. This allows the integration of the SXR emission along the path of the camera line of sight, yielding the time-dependent simulation of the SXR measurements for an arbitrary toroidal mode number and mode amplitude.

2.4. Determination of the LLM saturated amplitude

Fourier analysis of the SXR fluctuations shows that from its onset until several tens of milliseconds later, the LLM does not feature any significant \( n = 2 \) component. Although the Mirnov coils spectrogram in figure 1 does show an \( n = 2 \) component, these coils measure the magnetic fluctuations outside the plasma, in contrast to the SXR cameras, which are able to observe the plasma core. In addition, it is likely that the \( n = 2 \) component in figure 1 arises as a non-linear consequence of the \( n = 1 \) mode. The absence of \( n > 1 \) modes is also expected from the stability analysis of the LLM [37]: the \( n = 1 \) component is unstable at larger \( q_{\text{min}} \) than the \( n = 2 \) component, and since the \( q \) profile evolves downwards during the discharge, the \( n = 1 \) component is always the only unstable at the appearance of the LLM, which is the phase analysed here. Consequently, we only consider the \( n = 1 \) component in the rest of this study.

The SXR signals are simulated for different mode amplitudes and compared with the experimental data. Since the MHD perturbation does not affect the average SXR signal but only results in its variation, this comparison is based on the SXR fluctuations only. This method reduces the influence of parasitic SXR sources, as for example impurity line emission. The simulation having the lowest residuals with respect to the measurements is then considered a good estimate of the mode amplitude. (Should it be necessary to include the \( n = 2 \) mode in the analysis, the simulation would be carried out with various \( n = 1 \) and \( n = 2 \) amplitudes, as well as toroidal phases between these components). It is assumed here that, as the instability saturates, its structure remains identical to the linear one. This assumption, although quite strong, is likely to hold outside the resonant surfaces and inertial layer of the mode [34, 48].

3. Torque according to NTV theory

The saturated MHD structure is determined as described in section 2. The braking it induces is calculated using NTV theory. This theory is introduced in section 3.1 and a formulation applicable to MAST plasmas is presented in section 3.2.

3.1. NTV theory

NTV theory describes the damping of the plasma flow arising from the breaking of axisymmetry [42, 49]. The underlying mechanism is most easily understood in collisional plasmas where the dissipation of toroidal angular momentum is similar to that occurring during magnetic pumping [50]. The presence of the magnetic perturbation results in the distortion of the plasma’s flux tubes. Flux conservation prescribes parts of the flux tube with small cross-section to have a high magnetic field, hence a high perpendicular pressure by conservation of the first adiabatic invariant (the fluid cell contains the same particles over time, provided many collisions occur during its toroidal precession). If the collision time is short compared with the period of the fluid cell motion, constant total pressure on the flux surface indicates that the parallel pressure is low. Conversely, portions of the flux tube with large cross-section have a low magnetic field, low perpendicular pressure and high parallel pressure. Therefore, as the fluid cell travels across the distorted flux tube, it experiences an oscillation of parallel and perpendicular pressures as well as of flux tube’s shape. In such a system, the work done by
the parallel and perpendicular pressure vanishes because the pressure and shape oscillations
are in phase. The effect of collisions is, however, not instantaneous, and the oscillation of
pressures lags that of shape, causing an overall braking of the fluid cell. This mechanism is
sketched in figure 6. In the collisionless regime more relevant to tokamaks, the variation of
the toroidal field results in a drift of the particles trapped in banana orbits. This gives rise to a
radial current which exerts a \( j \times B \) torque on the plasma.

A quantitative expression for the NTV force can be obtained by solving the bounce
averaged drift-kinetic equation, then taking the velocity moment of the distribution function to
obtain the radial flux, and eventually using the flux-friction relation derived from neoclassical
theory [51, 52] to obtain the plasma viscosity. This derivation is carried out in [42].

NTV theory has been applied extensively to externally driven, static magnetic perturbation
cases [53, 54]. This theory can also be used if the field’s axisymmetry is broken by the
presence of an MHD instability, in which case the torque arises from the differential flow
of the plasma through the non-axisymmetric perturbation. In this case, the flow damping
brings the rotation of the plasma into agreement with that of the magnetic structure (figure 7).
The equivalence with the static case is found by moving from the lab frame to that travelling
with the MHD instability, a frame change which is possible only because the mode has a
rigid body rotation. The angular frequency of this motion results from the interaction of
the non-axisymmetric magnetic structure with both the plasma and the conducting external
components of the tokamak. This frequency is measured experimentally, so that it is not
necessary to calculate the external drag on the magnetic structure. This drag explains why the
presence of the internal MHD mode not only leads to angular momentum density redistribution
but also to an overall loss of momentum.

Since the magnetic perturbation is not applied externally, its structure has to be calculated
using an MHD code. Although there are some uncertainties inherent in this calculation, it is a

![](related_publications.png)

**Figure 6.** Heuristic mechanism of magnetic pumping. The top panel represents a distorted flux
tube cut open and straightened. The bottom panel shows the modulation of the parallel (plain line)
and perpendicular (dashed line) pressures caused by the shape modulation. The effect of collisions
not being instantaneous, the modulation of pressures lags that of shape. As the fluid cell moves
toroidally, the work of the perpendicular pressure during the first half of the cycle, which is in the
direction of the motion, (left) is less than that during the second half of the cycle, which opposes the
motion (right). Estimating the work of parallel pressure is more difficult since it needs to take into
account the pressure gradient, the variation of the cross-section and the length of the fluid element.
Nevertheless, it also opposes the motion.
Figure 7. The two competing angular frequency profiles in a rotating plasma with an MHD mode: that of the plasma (plain line) and that of the mode (dotted line). These profiles are brought in agreement by the damping of the plasma rotation, with a departure of the plasma angular frequency from that of the MHD of the order of the diamagnetic frequency, $\omega_{\text{NC}}$ (dashed line). This process is described by equation (5).

Figure 8. Validity of the $1/\nu$ regime for MAST shot 21508 at $t = 255$ ms. This regime is characterized by $q\omega_{E\times B} < v_i/\epsilon < \sqrt{\epsilon \omega_{t,i}}$, where $q\omega_{E\times B}$ is represented by the dotted line, $v_i/\epsilon$ by the solid line and $\sqrt{\epsilon \omega_{t,i}}$ by the dashed line.

self-consistent determination of the magnetic perturbation in contrast to that carried out in the externally applied field case. This self-consistency was recently shown to have a significant impact on the predicted NTV torque [55].

3.2. Formulation

MAST plasmas are mainly in the so-called $1/\nu$ collisionality regime (figure 8), where the particles trapped in banana orbits are collisionless and dominate the radial flux of ions. This
regime is characterized by $q \omega_{E \times B} < \nu_{ii} / \epsilon < \sqrt{\epsilon} \omega_{th,i}$ with $\omega_{E \times B}$ the $E \times B$ drift frequency, $\nu_{ii}$ the thermal ion collision frequency and $\epsilon$ the local aspect ratio. $\omega_{th,i} = (R_0 q)^{-1} v_{th,i}$ is the ion transit frequency, $v_{th,i}$ the thermal ion velocity and $R_0$ the major radius of the plasma.

The NTV theory is expressed in straight field line, constant Jacobian, Hamada coordinates $(v, \zeta, \theta)$ [56], with $v$ the volume enclosed by the flux surface, $\zeta$ and $\theta$ the toroidal and poloidal coordinates. From the usual geometrical flux coordinates $(\psi, \phi, \theta_g)$ (\psi being the poloidal flux, $\phi$ and $\theta_g$ the geometrical toroidal and poloidal angles), they can be obtained by the transformation [57]:

$$v(\psi) = 2\pi \int_0^\psi d\psi' \oint \frac{d\theta'_g}{B \cdot \nabla \theta'_g}, \quad (2a)$$

$$\zeta(\psi, \phi, \theta_g) = \frac{\phi}{2\pi} + F(\psi) \int_{\theta_g}^{\theta_g'} \left( \frac{1}{R^2} - \frac{1}{R^2} \right) \frac{d\theta'_g}{B \cdot \nabla \theta'_g}, \quad (2b)$$

$$\theta(\psi, \phi, \theta_g) = \left( \oint \frac{d\theta'_g}{B \cdot \nabla \theta'_g} \right)^{-1} \left( \int_{\theta_g}^{\theta_g'} \frac{d\theta'_g}{B \cdot \nabla \theta'_g} \right). \quad (2c)$$

Note that with the conventions chosen here, $\zeta$ and $\theta$ are periodic, of period unity (not $2\pi$). In equation (2b), $F(\psi) = R B_\phi$ with $B_\phi$ the toroidal field. Angled brackets denote a flux surface average carried out in the following manner:

$$\langle X \rangle = \left( \oint \frac{d\theta'_g}{B \cdot \nabla \theta'_g} \right)^{-1} \left( \oint X \frac{d\theta'_g}{B \cdot \nabla \theta'_g} \right). \quad (3)$$

Reference [57] details some properties of the coordinates built using equations (2a)–(2c) but does not demonstrate that they are the Hamada coordinates. This is done in appendix A. The braking depends on $|B|$, the modulus of the total magnetic field, which needs to be written in its Lagrangian form:

$$|B(X + \xi)| = |B_0 + \delta B + (\xi \cdot \nabla) B| = |B_0| \left( 1 + \sum_{(m,n)\neq(0,0)} \left( \frac{b_{m,n}}{|B_0|} \right) e^{2\pi i (m\theta - n\zeta)} \right). \quad (4)$$

Here, $X$, $\xi$ and $B_0$ are the position vector, the displacement vector and the equilibrium magnetic field, respectively. By construction, $b_{m,n}$ are the coefficients of the Fourier decomposition of the magnetic perturbation. In equation (4), the term $(\xi \cdot \nabla) B$ is most easily calculated by exploiting the equilibrium field’s axisymmetry and rewriting it as $(\xi \cdot \nabla) B = B_0(X + \xi) - B_0(X) + O(|\xi|^2)$. This calculation is detailed in appendix B and avoids tedious use of the tractable, though often diverging, $\nabla \times (\xi \times B)$ and $\nabla (\xi \cdot B)$ operators.

Rather than the expression given in [42], it is convenient to use the expression for the $1/v$ regime NTV torque given in [49], which makes use of the force balance equation for the ion fluid in order to express the torque as a function of the plasma profiles:

$$t_{\phi, NTV} = K \sum_{(m,n)\neq(0,0)} |n b_{m,n}|^2 W_{m,n} \left( (\omega_{\phi} - \omega_{NC}) - \omega_{MHD} \right) \quad (5)$$

with

$$K = 1.74 n_e T_e R B_{\phi} e^{3/2} \langle R \rangle \langle B_{\phi}^{-1} \rangle \langle R^{-2} \rangle. \quad (6)$$

Here, $T_i$, $B_{\phi}$, $R$ and $\omega_{\phi}$ are the ion fluid temperature, the toroidal magnetic field, the major radius and the toroidal angular frequency, respectively. The ion temperature and velocity are
assumed to be equal to those of the carbon fluid measured by CXRS. \( \omega_{\text{MHD}} \) is the angular frequency of the MHD, calculated by Fourier transform of the SXR signal, it accounts for the needed change from the lab frame to that of the MHD mode. The \( W_{m,n} \) coefficients are given by

\[
W_{m,n} = \int_0^1 \left( \frac{F_{mnc}(\kappa)}{E(\kappa) - (1 - \kappa^2)K(\kappa)} \right)^2 + \left( \frac{F_{mns}(\kappa)}{E(\kappa) - (1 - \kappa^2)K(\kappa)} \right)^2 \, d\kappa^2,
\]

where \( \kappa \) is a pitch-angle parameter defined in [42], \( E(\kappa) \) and \( K(\kappa) \) the complete elliptic integrals of first and second kind. \( F_{mnc}(\kappa) \) and \( F_{mns}(\kappa) \) are defined by

\[
F_{mnc}(\kappa) = 2 \int_0^{2\arcsin(\kappa)} \sqrt{\kappa^2 - \sin^2(\theta/2)} \cos((m - nq)\theta) \, d\theta,
\]

\[
F_{mns}(\kappa) = 2 \int_0^{2\arcsin(\kappa)} \sqrt{\kappa^2 - \sin^2(\theta/2)} \sin((m - nq)\theta) \, d\theta.
\]

\( \omega_{\text{NC}}^* \) is a neoclassical offset angular frequency defined as

\[
\omega_{\text{NC}}^* = \frac{3.5}{Ze\bar{R}B_p} \frac{dT_i}{dr},
\]

where \( B_p \) and \( r \) are the poloidal field and minor radius. \( \bar{R} \) is the major radius of outboard mid-plane point of the flux surface. This offset rotation has been shown to be of importance [58].

The NTV torque given by equation (5) is proportional to the squared amplitude of the magnetic perturbation, and to the difference of the ion fluid and mode’s frequencies, \( \omega_{\phi} - \omega_{\text{MHD}} \) (with an additional offset \( \omega_{\text{NC}}^* \)), as expected from the heuristic mechanisms introduced in section 3.1.

4. MAST results

The comparison described in the previous sections and summarized in figure 2 was carried out on MAST plasmas featuring the LLM, a saturated ideal-MHD mode. The eigenstructure calculated by CASTOR, for an equilibrium reconstructed at the LLM appearance, is an \( n = 1 \) internal kink mode, shown in figure 9. The best agreement between SXR simulations and experimental data is obtained for a radial amplitude \( \xi_r = 1.2 \) cm. The relative amplitudes as well as the phases of the SXR fluctuations are well matched for each channel (figures 10 and 11). This best match corresponds to a clear local minimum of the simulation’s residuals with respect to the measured data (figure 12), which gives good confidence in the estimated amplitude. As mentioned in section 2, no \( \pi \) phase jumps are observed in the SXR fluctuations (figure 11), tending to rule out electromagnetic torques associated with magnetic reconnection.

The rotation frequency profiles of the plasma during the braking are shown in figure 13. The rotation is unchanged for the radial location \( R = 1.15 \) m. It is tempting to interpret this point as the position where the mode’s frequency and that of the plasma are equal, hence where one would expect the torque applied by the mode to vanish. This is actually not the case, and the dashed line in figure 13 indicates the frequency of the mode at its onset. Although it decreases on equilibrium timescales, this latter frequency does not reach that of the plasma at \( R = 1.15 \) m at any time. This gap between the rotation frequency of the plasma at \( R = 1.15 \) m and that of the mode can be explained by the presence of the offset frequency \( \omega_{\text{NC}}^* \) in the NTV formulation (equation (5)), and these two shifts are of similar magnitude.

When the LLM appears, the plasma is assumed in a steady state from the point of view of angular momentum transport. This means that the angular momentum input from NBI
exactly balances momentum transport and losses. The braking of the plasma takes place on time scales faster than the momentum confinement time (\(\sim 50\) ms on MAST), such that the NBI source can still be assumed to balance the momentum transport and losses during the first milliseconds of the braking: over this period, the latter is solely due to MHD. The comparison carried out in this study focuses on this early rotation damping. Analysing later time slices would require an additional assumption on the transport of angular momentum, which cannot confidently be made. Note, however, that the LLM increases fast ion losses, possibly leading to two phenomena neglected here: the torque deposition may be slightly altered after mode onset and the fast ion redistribution is likely to affect plasma rotation. Initial calculations of this second effect, however, indicate that it is significantly lower than the observed damping. More accurate calculations using the HAGIS drift-kinetic code are to be carried out in the future.

The torque predicted by NTV for MAST discharge 21508 at \(t = 255\) ms is plotted in figure 14, together with the measured rate of change of angular momentum for each flux tube. The predictions and observations have the same order of magnitude. The profile shapes are similar, except in the vicinity of the rational surfaces and the inertial layer, the latter being located at the \(q_{\text{min}}\) surface, due to the absence of a \(q = 1\) surface. In these regions, large parallel magnetic field perturbations result in a high torque which is not observed in the experimental data. Since the linear structure used here only differs from the non-linear saturated one at these positions [34, 48], this disagreement is not regarded as invalidating the applicability of the theory to the observations. The uncertainties in the different plasma profiles involved in the calculation are bounded at a level that does not compromise the calculated order of magnitude.
5. Conclusions

MAST NBI-heated plasmas with low density exhibit a weakly reversed shear $q$ profile just above 1, prone to the appearance of the LLM, a saturated long-lived MHD instability. All experimental observations indicate that the mode is ideal in nature, which is corroborated by analytical work and modelling [37]. Measurements show strong core damping of the sheared toroidal rotation simultaneous to the mode onset. The eigenstructure of the mode was calculated using the CASTOR code, and its saturated amplitude determined using the fluctuations of SXR experimental data. The structure of the magnetic perturbation obtained was used to estimate magnitude of the results, nor the shape of the profile. Nevertheless, they do not allow more detailed comparisons, and seem not accurate enough to have a predictive use. It is, however, worth mentioning that the inclusion of the offset frequency $\omega_{NC}$ is crucial in order to reproduce the measured torque profile with NTV theory. Furthermore, the Lagrangian term $(\xi \cdot \nabla)B$ must be taken into account to predict a torque of magnitude comparable to the one observed experimentally, neglecting it decreases the calculated result by up to 70%, especially in the vicinity of the magnetic axis.

Figure 10. The experimental fluctuations caused by the MHD mode on MAST SXR horizontal array for shot 21508 at $t = 255$ ms, and the simulated signal based on a radial mode amplitude of 1.2 cm.
Figure 11. Relative phases of the simulated SXR chords (plain line) and those of the experimental ones (dashed line). Chords U9 to L1 observe from the bottom of the plasma to the mid-plane, while chords L9 to L2 observe from the top of the plasma to the mid-plane. Chord pairs (L3, U3) to (L7, U7) show a constant phase, which is consistent with the kink eigenstructure predicted by CASTOR.

Figure 12. Residuals from the comparison between the SXR experimental measurements and the simulations for a range of perturbation amplitudes. The graph clearly shows optimal agreement for a radial amplitude of $\xi_r = 1.2$ cm.

the braking torque induced by the mode according to NTV theory, the rationale of this entire study being summarized in figure 2. The results were found in agreement with the experiment, in terms of order of magnitude and profile.

In previous work, NTV has mainly been applied to externally applied magnetic perturbations [53, 54]. The similarities between predictions and experimental observations are encouragements that this theory is also a good candidate mechanism for the interaction
between MHD and plasma rotation, as described in section 3. For this type of study, the theory is applied in the frame moving with the MHD mode, which has a rigid body rotation. This application to an MHD instability also provides an additional observation of the offset rotation term $\omega^{*}_{NC}$, after that made in experiments with coil-induced magnetic perturbations on DIII-D [58].

This study of the interplay between an MHD mode and rotation is not limited to MAST cases, but is relevant to several tokamaks exploiting scenarios with hybrid-like, reversed shear $q$ profiles just above an integer value. Ideal saturated $(m,n) = (2, 1)$ modes are observed in JET’s hybrid plasmas with $q_{\text{min}} > 2$ [59, 60]. This so-called continuous mode also results in a flattening of core rotation. Slowly growing ideal modes associated with hybrid like $q$ profile
and high normalized pressure have also been observed on DIII-D [61] and JT-60U [62], with a simultaneous collapse of the toroidal rotation. Damping of core rotation by MHD has also been observed in NSTX [63], although the mode was observed to be resistive and the braking attributed to electromagnetic torques arising from magnetic reconnection.

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Appendix A. Hamada coordinates

The Hamada coordinates \((v, \zeta, \theta)\) [56] have the following properties.

Property 1. The first coordinate is the volume enclosed by the flux surfaces.

Property 2. They are straight field line coordinates, meaning that \((B \cdot \nabla \zeta)(B \cdot \nabla \theta)^{-1} = q(v)\) and this ratio is equal to the usual safety factor, which in usual geometrical flux coordinates is \(q(v) = (2\pi)^{-1} \oint (B \cdot \nabla \phi)(B \cdot \nabla \theta_g)^{-1} d\theta_g\).

Property 3. All the contravariant components of the magnetic field are flux functions.

Property 4. Their Jacobian is constant and unity.

These properties uniquely define the Hamada covariant basis, hence the full coordinates, since \(\nabla v\) is fully determined by property 1; properties 2 and 3 set the relative angles and magnitudes of \(\nabla \zeta\) and \(\nabla \theta\) and lastly, property 4 sets the relative angles and magnitudes of \(\nabla v\) on the one hand and \((\nabla \zeta, \nabla \theta)\) on the other hand. It is shown in this section that the coordinates \((v, \zeta, \theta)\), given by equations (2a)–(2c), satisfy these four properties and therefore are the Hamada coordinates.

It should first be mentioned that the integral \(\oint X (B \cdot \nabla \theta_g)^{-1} d\theta_g\) is left unchanged by any monotonic periodic variable change:

\[
\theta'_g = f(\eta) \Rightarrow \begin{cases} d\theta'_g = f'(\eta) d\eta \\ \nabla\theta'_g = f'(\eta) \nabla\eta \Rightarrow \oint X (B \cdot \nabla\theta'_g)^{-1} d\theta'_g = \oint X (B \cdot \nabla\eta)^{-1} d\eta. \end{cases}
\]

While not directly relevant to this demonstration, this property interestingly shows that any monotonic periodic poloidal coordinate can be used in place of \(\theta_g\) to build the \((v, \zeta, \theta)\) coordinates.

Property 1 is a simple matter of definition. To prove the validity of properties 2–4, it is useful to note a few points beforehand. To begin with, the definition of the volume in equation (2a) comes from the basic properties of the magnetic field, allowing one to express it as \(B = F(\psi) \nabla \phi + \nabla \psi \times \nabla \phi\). Since \(\nabla \theta_g \cdot \nabla \phi = 0\), this gives

\[
dV = d\psi d\phi d\theta_g = \nabla \psi \cdot (\nabla \phi \times \nabla \theta_g) = \nabla \theta_g \cdot (\nabla \psi \times \nabla \phi) = \frac{d\psi d\phi d\theta_g}{B \cdot \nabla \theta_g}.
\]
and therefore
\[ v(\psi) = 2\pi \int_0^\psi d\psi' \oint \frac{d\theta_g'}{B \cdot \nabla \theta_g'}. \]  
(A.5)

In addition, the third covariant vector of the \((v, \zeta, \theta)\) coordinates is
\[ \nabla \theta = \frac{\partial \theta}{\partial \psi} \nabla \psi + \frac{\partial \theta_g}{\partial \theta_g} \nabla \theta_g \]  
(A.6)

Hence
\[ B \cdot \nabla \theta = \frac{\partial \theta}{\partial \psi} B \cdot \nabla \psi + \left( \oint \frac{d\theta_g}{B \cdot \nabla \theta_g} \right)^{-1} B \cdot \nabla \theta_g. \]  
(A.7)

This means successively
\[ \nabla \theta_g \frac{1}{B \cdot \nabla \theta_g} = \nabla \theta \frac{1}{B \cdot \nabla \theta} + \frac{\partial \theta_g}{\partial \psi} \frac{\nabla \psi}{B \cdot \nabla \theta_g}. \]  
(A.11)

Lastly, the covariant vectors of the \((v, \zeta, \theta)\) coordinates are given by simple differentiation, and use of equation (A.9):
\[ \nabla v = \frac{2\pi}{B \cdot \nabla \theta} \nabla \psi, \]  
(A.13)

\[ \nabla \zeta = \frac{\partial \zeta}{\partial \psi} \nabla \psi + \frac{1}{2\pi} \nabla \phi + \frac{F(\psi)}{2\pi} \left( \frac{1}{R^2} \right) \frac{1}{B \cdot \theta_g}. \]  
(A.14)

\[ \nabla \theta = \frac{\partial \theta}{\partial \psi} \nabla \psi + B \cdot \nabla \theta \frac{\nabla \theta_g}{B \cdot \nabla \theta_g}. \]  
(A.15)

With these remarks in mind, it is possible to easily prove properties 2 to 4.

By definition, \(B \cdot \nabla v = 0\), and equation (A.9) indicates that \(B \cdot \nabla \theta\) is a flux function. Taking the scalar product of \(B\) with equation (A.14) results in
\[ B \cdot \nabla \zeta = \frac{1}{2\pi} B \cdot \nabla \phi + \frac{F(\psi)}{2\pi} B \cdot \nabla \theta_g \left( \frac{1}{R^2} \right) \frac{1}{B \cdot \theta_g}. \]  
(A.16)

Since by definition of \(F(\psi)\) and \(\phi\), \(B \cdot \nabla \phi = F(\psi) R^{-2}\), we have
\[ B \cdot \nabla \zeta = \frac{F(\psi)}{2\pi} \left( \frac{1}{R^2} \right) \]  
(A.17)

This equation guarantees that \(B \cdot \nabla \zeta\) is the flux surface average of \(B \cdot \nabla \phi\). Consequently, all contravariant components of the magnetic field are flux functions and property 3 is verified.
Substituting for $B \cdot \nabla \phi = R^{-2} F(\psi)$ in the safety factor definition gives

$$q(v) = \frac{1}{2\pi} \oint \frac{F(\psi)/(2\pi R^2)}{B \cdot \nabla \theta_g} \, d\theta_g$$

(A.18)

$$= \frac{F(\psi)}{2\pi} \oint \frac{d\theta_g/R^2}{B \cdot \nabla \theta_g}.$$  

(A.19)

Now using the definition of the flux surface average (equation (3)) together with equations (A.17) and (A.9):

$$q(v) = \left( \oint \frac{d\theta_g}{B \cdot \nabla \theta_g} \right) B \cdot \nabla \zeta$$

(A.20)

$$= \frac{B \cdot \nabla \zeta}{B \cdot \nabla \theta}.$$  

(A.21)

This demonstrates that property 2 is verified.

There only remains to prove property 4. This is done by calculating the Jacobian of the coordinates $(v, \zeta, \theta)$. Taking the cross-product of equations (A.13) and (A.14) gives

$$\nabla v \times \nabla \zeta = \frac{1}{B \cdot \nabla \theta} \left( \nabla \psi \times \nabla \phi + F(\psi) \left( \frac{1}{R^2} - \frac{1}{R^2} \right) \nabla \psi \times \nabla \theta_g \right).$$  

(A.22)

Using equation (A.12) to transform the second term on the right-hand side:

$$\nabla v \times \nabla \zeta = \frac{1}{B \cdot \nabla \theta} \left( \nabla \psi \times \nabla \phi + F(\psi) \left( \frac{1}{R^2} - \frac{1}{R^2} \right) \nabla \psi \times \nabla \theta \right).$$  

(A.23)

Taking the scalar product of equations (A.23) with $\nabla \theta$, the second term on the right-hand side vanishes leaving

$$(\nabla v \times \nabla \zeta) \cdot \nabla \theta = \frac{1}{B \cdot \nabla \theta} ((\nabla \psi \times \nabla \phi) \cdot \nabla \theta).$$  

(A.24)

Substituting for $\nabla \theta$ on the right-hand side using equation (A.15) then yields

$$(\nabla v \times \nabla \zeta) \cdot \nabla \theta = \frac{1}{B \cdot \nabla \theta} \left( (\nabla \psi \times \nabla \phi) \left( \frac{\partial \theta}{\partial \psi} \nabla \psi + B \cdot \nabla \theta \cdot \nabla \theta_g \right) \right)$$

(A.25)

$$= (\nabla \psi \times \nabla \phi) \cdot \frac{\nabla \theta_g}{B \cdot \nabla \theta_g}.$$  

(A.26)

Using equations (A.4) and (A.26) eventually proves property 4:

$$(\nabla v \times \nabla \zeta) \cdot \nabla \theta = \frac{(\nabla \psi \times \nabla \phi) \cdot \nabla \theta_g}{B \cdot \nabla \theta_g}$$

(A.27)

$$= \frac{B \cdot \nabla \theta_g}{B \cdot \nabla \theta_g}$$

(A.28)

$$= 1.$$  

(A.29)

The $(v, \zeta, \theta)$ coordinates built using equations (2a)–(2c) therefore verify properties 1–4, and thus are the Hamada coordinates. Their construction is closely linked to the expression of the volume given by equation (A.4) and the flux surface average defined in equation (3).
Appendix B. Calculation of $(\xi \cdot \nabla) B$

NTV theory requires the magnetic perturbation to be expressed in the Lagrangian form. The Lagrangian term to be added to the Eulerian form of $\delta B$ is simply the contribution of the displacement of the fluid cell to the perturbation, $(\xi \cdot \nabla) B = B_0(X + \xi) - B_0(X) + O(|\xi|^2)$.

To carry out this calculation, the equilibrium magnetic field is most conveniently decomposed on the usual cylindrical local basis $(e_R, e_\phi, e_Z)$, but with the use of the $(\psi, \phi, \theta_g)$ coordinates to locate a point in space:

$$B_0(X) = B_{0,R}(\psi, \theta_g)e_R + B_{0,\phi}(\psi, \theta_g)e_\phi + B_{0,Z}(\psi, \theta_g)e_Z.$$  \hspace{1cm} (B.1)

According to basic differential geometry, the equilibrium field at $X + \xi$ is

$$B_0(X + \xi) = B_0(X) + \frac{\partial B_0}{\partial \psi}(\nabla \psi \cdot \xi) + \frac{\partial B_0}{\partial \phi}(\nabla \phi \cdot \xi) + \frac{\partial B_0}{\partial \theta_g}(\nabla \theta_g \cdot \xi) + O(|\xi|^2).$$  \hspace{1cm} (B.2)

Therefore

$$(\xi \cdot \nabla) B = \frac{\partial B_0}{\partial \psi}\xi^\psi + \frac{\partial B_0}{\partial \phi}\xi^\phi + \frac{\partial B_0}{\partial \theta_g}\xi^{\theta_g} + O(|\xi|^2).$$  \hspace{1cm} (B.3)

The differential forms of $B_0$ must be calculated taking into account that the field is axisymmetric and that the chosen basis vectors are not constant in space but depend on the toroidal angle. This gives, for $\alpha \in (\psi, \theta_g)$:

$$\frac{\partial B_0}{\partial \alpha} = \frac{\partial B_{0,R}}{\partial \alpha}e_R + \frac{\partial B_{0,\phi}}{\partial \alpha}e_\phi + \frac{\partial B_{0,Z}}{\partial \alpha}e_Z$$  \hspace{1cm} (B.4)

and

$$\frac{\partial B_0}{\partial \phi} = B_{0,R}e_R + \frac{\partial B_{0,\phi}}{\partial \phi}e_\phi + \frac{\partial B_{0,Z}}{\partial \phi}e_Z - B_{0,R}e_\phi - B_{0,\phi}e_R.$$  \hspace{1cm} (B.5)

Equations (B.3), (B.4), (B.5) allow a simple calculation of the Lagrangian term of the magnetic perturbation, avoiding the intricate, although widespread, use of the $\nabla \times (\xi \times B)$ and $\nabla (\xi \cdot B)$ operators. Each of these individually diverge, but their diverging parts cancel each other in the combination involved in $(\xi \cdot \nabla) B$. The Lagrangian term is given in its simple form by

$$(\xi \cdot \nabla) B = \xi^\psi \left( \frac{\partial B_{0,R}}{\partial \psi}e_R + \frac{\partial B_{0,\phi}}{\partial \psi}e_\phi + \frac{\partial B_{0,Z}}{\partial \psi}e_Z \right) + \xi^\phi \left( B_{0,R}e_\phi - B_{0,\phi}e_R \right) + \xi^{\theta_g} \left( \frac{\partial B_{0,R}}{\partial \theta_g}e_R + \frac{\partial B_{0,\phi}}{\partial \theta_g}e_\phi + \frac{\partial B_{0,Z}}{\partial \theta_g}e_Z \right) + O(|\xi|^2).$$  \hspace{1cm} (B.6)

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